NUSC Technical Report 8595
10 August 1989

Operating Characteristics for Combiner with a Dead Zone in Each Channel

Albert H. Nuttall
Surface ASW Directorate

Naval Underwater Systems Center
Newport, Rhode Island / New London, Connecticut

Preface

This research was conducted under NUSC Project No. A75205, Subproject No. RR0000-N01, "Applications of Statistical Communication Theory to Acoustic Signal Processing," Principal Investigator Dr. Albert H. Nuttall (Code 304). This technical report was prepared with funds provided by the NUSC In-House Independent Research and Independent Exploratory Development Program, sponsored by the Office of Chief of Naval Research. Also, this research was conducted under Project No. PE6533N, "Surface Ship ASW Advanced Development," Principal Investigator Ira B. Cohen (Code 33A), Project Manager David M. Ashworth (Code 33A), sponsored by NAVSEA, Program Managers CDR L. Schneider and Eric Plummer (Code 63D).

The technical reviewer for this report was Ira B. Cohen (Code 33A).

Reviewed and Approved: 10 August 1989

E. J. Sullivan

Daniel M. Viccione
Associate Technical Director
Research and Technology

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) TR 8595	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Underwater Systems Center; 6b. OFFICE SYMBOL (If applicable) Code 304	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) New London Laboratory New London, CT 06320	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Chief of Naval Research; 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. — ; PROJECT NO. A75205; TASK NO. — ; WORK UNIT ACCESSION NO. —

11 TITLE (Include Security Classification)

OPERATING CHARACTERISTICS FOR COMBINER WITH A DEAD ZONE IN EACH CHANNEL

12. PERSONAL AUTHOR(S)

A. H. Nuttall

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
	FROM ____ TO ____	1989 August 10	60

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Operating Characteristics, Combiner, Receiver Performance, Dead Zone, Matched Filters, Detection Probability

19 ABSTRACT (Continue on reverse if necessary and identify by block number) Cont'd

The receiver operating characteristics, namely detection probability versus false alarm probability, for a combiner which employs nonlinearities with dead zones in each of its channels, are derived in closed form, as a function of: N, the number of channels; F, the fraction of data passed by the nonlinearity in each channel; R, the signal-to-noise ratio in each channel; and T, the system output threshold. Plots of these results for N = 1, 2, 4, 6, 8, 16, 32, 64 and F = 1, .1, .01, .001 reveal that inclusion of the dead zone does not significantly degrade performance, the typical loss being of the order of 1 dB for small N, and 3 dB for larger N, in the important operating ranges. The only limitation is that certain ranges of false alarm probabilities are unachievable; however, since these ranges generally correspond to undesirable operating conditions, the limitation is not too relevant. Keywords,

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL A. H. Nuttall	22b. TELEPHONE (Include Area Code) (203) 440-4618; 22c. OFFICE SYMBOL Code 304

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted.
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

18. SUBJECT TERMS (Cont'd.)

False Alarm Probability
Breakpoint of Threshold.
Nonlinearity
Multiple Channels

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (cont'd)

LIST OF TABLES

LIST OF SYMBOLS

N	number of channels, figure 1
F	fraction of data passed by nonlinearity, (3),(27)
R	signal-to-noise power ratio in each channel
T	system output threshold, figure 1
r_n	n-th input to system, figure 1
x_n	squared-envelope filter output, figure 1
L	breakpoint (threshold) of nonlinearity, figure 2,(28)
y_n	output of nonlinearity, figure 1
z	system output, figure 1
$p_x(u)$	probability density function of random variable x
$P_x(u)$	cumulative distribution function of x, (1),(13)
$Q_x(u)$	exceedance distribution function of x, (1),(13)
$p_y(u)$	probability density function of random variable y, (4)
$f_y(\xi)$	characteristic function of random variable y, (5),(14)
z	summer output, (6)
$f_z(\xi)$	characteristic function of random variable z, (7),(16)
P_F	false alarm probability, (8),(26),(32)
P_D	detection probability, (8),(25),(32)
a	auxiliary parameter, (12)
B	auxiliary parameter, (15)
$E(u,n)$	auxiliary function, (17)
$p_n(u)$	normalized probability density function, (18)
$f_n(\xi)$	normalized characteristic function, (19)
$Q_n(u)$	normalized exceedance distribution function, (20)

LIST OF SYMBOLS (cont'd)

$p_z(u)$	probability density function of output z, (22)
$Q_z(u)$	exceedance distribution function of z, (23)
R(dB)	signal-to-noise ratio R in decibels, (33)
ROC	receiver operating characteristic

OPERATING CHARACTERISTICS FOR COMBINER WITH A DEAD ZONE IN EACH CHANNEL

INTRODUCTION

Some data processing shortcuts are often required in order to keep the computational burden in today's detection and tracking systems within manageable limits. One strategem to accomplish this goal is to quantize the signal levels at various points in the receiver processing chain. Another is to reject low-level quantities, and retain only the larger terms, in the belief that only the latter will lead to statistically meaningful decisions on signal presence versus absence.

Here, we investigate a ~~one such~~ technique, where all levels below a breakpoint or threshold value are rejected, that is, set to zero, while those signal levels above the breakpoint are retained in their full accuracy. In particular, this approach is employed in each branch of a combiner, as encountered in diversity or multiple ping transmission. The question to be addressed is the cost of this data reduction procedure, in terms of the additional signal-to-noise ratio required to maintain a desired level of performance, as measured by the false alarm and detection probabilities.

Cont'd on DD 1473

PROCESSOR DESCRIPTION

The processor of interest is depicted in figure 1. Received inputs $r_1, \ldots, r_N$ are either composed of noise-only or they all contain signal plus noise. An example of this situation is afforded by a multiple ping transmission, with search on the range of a possible target. The received signal in each channel (if present) is match-filtered and square-law envelope-detected at the candidate time instant of suspected or hypothesized peak output.

At this point, instead of simply summing up these multiple outputs, and in an effort to reduce the amount of information sent on for further data processing, the squared envelope x_n in the n-th channel is subjected to the nonlinear operation depicted in figure 2. Namely, all input levels to the nonlinearity below breakpoint (threshold) value L are replaced by zero, whereas those levels above the breakpoint are kept as is. The breakpoint value L is chosen so that a specified fraction F of the input data to the nonlinearity is passed, when noise-alone is present at the inputs; the hope is that F can be chosen very small, without significant degradation in performance. Finally, the output of the summer in figure 1 is compared with output threshold T for purposes of deciding on signal presence versus absence.

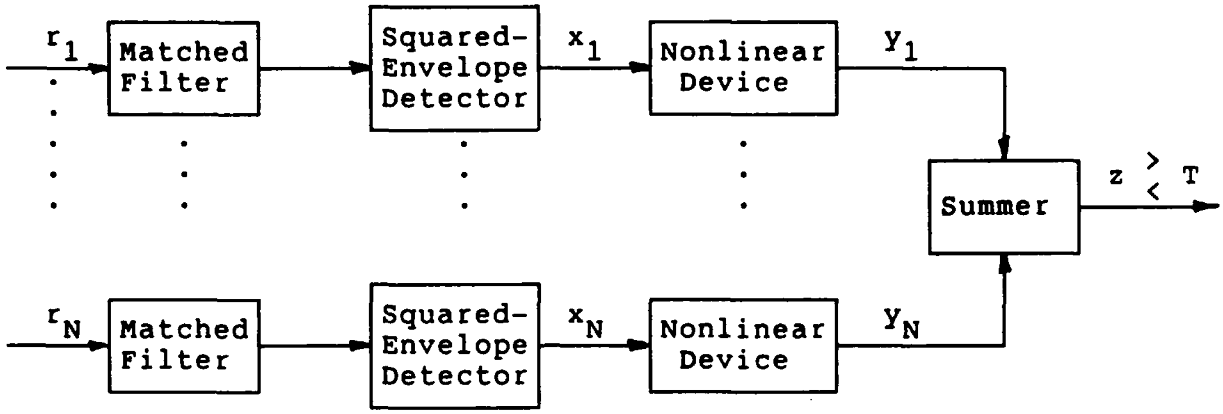

Figure 1. Processor Block Diagram

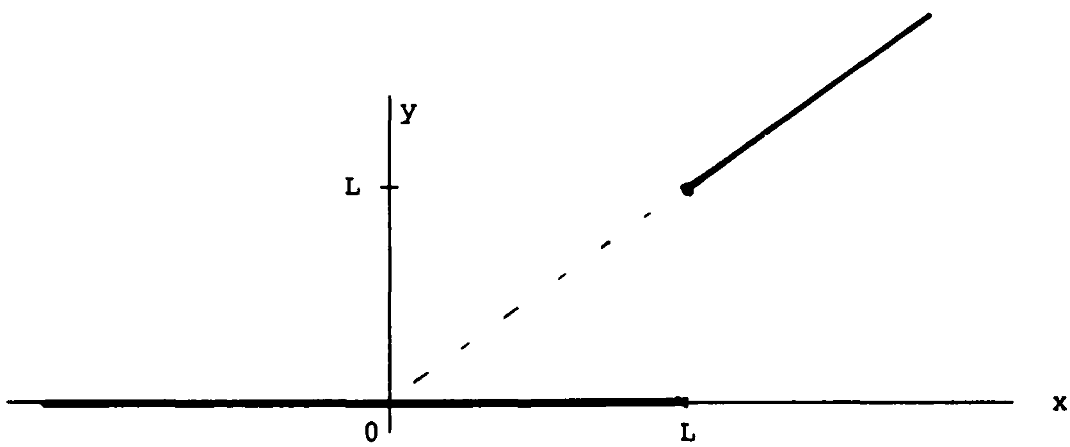

Figure 2. Nonlinear Device Characteristic

ANALYSIS OF PERFORMANCE

The inputs $\{r_n\}$ to figure 1 are presumed to be statistically independent of each other, whether signal is present or not. The squared-envelope outputs $\{x_n\}$ are, therefore, also statistically independent of each other, with probability density function $p_x(u)$, which is presumed known for both cases of signal present as well as signal absent. The corresponding cumulative distribution function and exceedance distribution function are, respectively,

$$P_x(u) = \int_{-\infty}^{u} dt\ p_x(t) = \mathrm{Prob}(x \le u),$$

$$Q_x(u) = \int_{u+}^{\infty} dt\ p_x(t) = \mathrm{Prob}(x > u). \tag{1}$$

The nonlinear device in figures 1 and 2 is characterized mathematically by

$$y = \begin{Bmatrix} 0 \text{ for } x < L \\ x \text{ for } x \ge L \end{Bmatrix}; \qquad L \ge 0. \tag{2}$$

Breakpoint L is presumed nonnegative, since the output of the squared-envelope detector in figure 1 can never be negative. The fraction of data passed by the nonlinearity is

$$F = \mathrm{Prob}(x > L) = Q_x(L). \tag{3}$$

Since exceedance distribution function Q_x is known, this equation can be solved for the required breakpoint value L, once fraction

F is specified. This calculation is done for the noise-only case, since the breakpoint is desired to be set for this condition.

Inspection of figure 2 immediately reveals that the probability density function of random variable y_n is given by

$$p_y(u) = P_x(L)\ \delta(u) + p_x(u)\ U(u - L), \tag{4}$$

where δ and U are the delta function and the unit step function, respectively. Therefore, the characteristic function of random variable y_n is

$$f_y(\xi) = \int_{-\infty}^{\infty} du\ \exp(i\xi u)\ p_y(u) =$$

$$P_x(L) + \int_{L}^{\infty} du\ \exp(i\xi u)\ p_x(u)\ . \tag{5}$$

Finally, using the statistical independence of the system inputs, the summer output,

$$z = \sum_{n=1}^{N} y_n\ , \tag{6}$$

has characteristic function

$$f_z(\xi) = \left[f_y(\xi)\right]^N = \left[P_x(L) + \int_{L}^{\infty} du\ \exp(i\xi u)\ p_x(u)\right]^N. \tag{7}$$

For general given probability density function $p_x(u)$, the integral on u in (7) can be done efficiently by means of a fast Fourier transform. Then the numerical evaluation of the exceedance distribution function of z, namely $Q_z(u)$, can be accomplished by the techniques utilized in [1, 2, 3]. This numerical approach would have to be carried out for both cases of signal absent and signal present, in order to get the false alarm probability P_F as well as the detection probability P_D. Specifically,

$$P_F = Q_z(T;\ \text{noise-only}),$$

$$P_D = Q_z(T;\ \text{signal-plus-noise}). \tag{8}$$

In essence, (7) characterizes the performance of the processor in figures 1 and 2. The remaining effort is the analytical and numerical manipulation of (7) into useful computer forms and evaluation.

EXPONENTIAL EXAMPLE

STATISTICS OF DETECTOR OUTPUT

If the inputs $\{r_n\}$ to the processor of figure 1 are Gaussian, then the squared-envelope detector outputs $\{x_n\}$ are exponentially distributed. We take the probability density function of x_n to be

$$p_x(u) = \left\{ \begin{array}{ll} 0 & \text{for } u<0 \\ \exp(-u) & \text{for } u\geq 0 \end{array} \right\} \text{ for noise-only.} \tag{9}$$

This corresponds to a mean value of

$$\overline{x} = \int_{-\infty}^{\infty} du\ u\ p_x(u) = 1 \qquad \text{for noise-only.} \tag{10}$$

This choice of scaling at the detector output does not constitute any loss of generality, since absolute level obviously has no effect upon the receiver operating characteristics of the processor in figure 1.

For Gaussian signal also present at the system input, the probability density function of x_n is

$$p_x(u) = \left\{ \begin{array}{ll} 0 & \text{for } u<0 \\ a\exp(-au) & \text{for } u\geq 0 \end{array} \right\} \text{ for signal-present.} \tag{11}$$

Here,

$$a = \frac{1}{1 + R}\ , \tag{12}$$

where R is the signal-to-noise power ratio at the matched filter output. (If R = 0, then a = 1, and (11) reduces to (9).) Thus, any signal processing gains associated with the filtering process are incorporated in the value of R. Observe that R is the signal-to-noise power ratio per channel or per ping, not the "total signal-to-noise ratio" at the system output.

Another signal model, which also leads to probability density function (11) for the detector output, is slow Rayleigh fading in the medium through which the transmitted pings traveled. That is, during a single ping duration, the medium attenuation is constant, but from ping to ping, the attenuation is statistically independent and governed by a Rayleigh probability density function on the received signal envelope.

CHARACTERISTIC FUNCTION OF OUTPUT z

We will determine the statistics of output z of figure 1 for the signal-present probability density function of x, as given by (11). The case for noise-only will then follow immediately by setting $a = 1$.

The cumulative distribution and exceedance distribution functions of x_n are given by substitution of (11) in (1), that is

$$P_x(u) = \begin{Bmatrix} 0 & \text{for } u<0 \\ 1 - \exp(-au) & \text{for } u\geq 0 \end{Bmatrix},$$

$$Q_x(u) = \begin{Bmatrix} 1 & \text{for } u<0 \\ \exp(-au) & \text{for } u\geq 0 \end{Bmatrix}. \tag{13}$$

The characteristic function of random variable y is obtained by substituting (11) and (13) in (5); thus

$$f_y(\xi) = 1 - \exp(-aL) + \int_L^{\infty} du\ a \exp(i\xi u - au) =$$

$$= 1 - B + B\,\frac{a \exp(i\xi L)}{a - i\xi}\ , \tag{14}$$

where we define

$$B = \exp(-aL); \quad L \geq 0. \tag{15}$$

The characteristic function of output z is given by (7) as

$$f_y(\xi) = \left[1 - B + B\,\frac{a \exp(i\xi L)}{a - i\xi}\ \right] =$$

$$= \sum_{n=0}^{N} \binom{N}{n} (1-B)^{N-n}\ B^n\ \frac{\exp(i\xi Ln)}{(1-i\xi/a)^n}\ . \tag{16}$$

AUXILIARY FUNCTIONS

Define the set of functions

$$E(u,n) = \left\{ \begin{array}{c} 1 \quad \text{for } u < 0 \\ \int_u^\infty dt \, \frac{t^n \exp(-t)}{n!} = \exp(-u) \; e_n(u) = \exp(-u) \sum_{k=0}^{n} \frac{u^k}{k!} \\ \text{for } u \geq 0 \end{array} \right\} \quad (17)$$

for $n \geq 0$. Here, we used the partial-exponential notation $e_n(u)$ given in [4; 6.5.11]. The expansion of the integral in (17) may be verified by repeated integrations by parts.

Also, define the set of normalized probability density functions

$$p_n(u) = \left\{ \begin{array}{ll} 0 & \text{for } u < 0 \\ \frac{u^{n-1} \exp(-u)}{(n-1)!} & \text{for } u \geq 0 \end{array} \right\} \quad \text{for } n \geq 1,$$

$$p_o(u) = \delta(u) \qquad \text{for all } u. \qquad (18)$$

The corresponding characteristic functions are

$$f_n(\xi) = \frac{1}{(1-i\xi)^n} \qquad \text{for } n \geq 0, \qquad (19)$$

while the exceedance distribution functions are

$$Q_n(u) = E(u,n-1) \quad \text{for all } u,\ n \geq 1,$$

$$Q_o(u) = \left\{ \begin{array}{l} 1 \text{ for } u < 0 \\ 0 \text{ for } u \geq 0 \end{array} \right\} . \qquad (20)$$

EXCEEDANCE DISTRIBUTION FUNCTION OF OUTPUT z

Since $p_n(u)$ and $f_n(\xi)$ are a Fourier transform pair for $n \geq 0$, it follows that

$$\frac{1}{(1-i\xi/a)^n} \quad \text{and} \quad a\, p_n(au) \tag{21}$$

are a Fourier transform pair. Then (16) allows us to determine the probability density function of output random variable z as

$$p_z(u) = \sum_{n=0}^{N} \binom{N}{n} (1-B)^{N-n} B^n \, a\, p_n(a(u - Ln)) \quad \text{for all } u, \tag{22}$$

where the "shift factor" $u - Ln$ is due to the $\exp(i\xi Ln)$ term. This is a useful expansion, even for large N, since all the terms are positive or zero; there is no cancellation, as there would be for an alternating series.

The exceedance distribution function of z follows immediately from (22) as

$$Q_z(u) = \sum_{n=0}^{N} \binom{N}{n} (1-B)^{N-n} B^n \, Q_n(a(u - Ln)) \quad \text{for all } u. \tag{23}$$

Again, this series has no negative terms. Also, the Q_n terms are sums of positive quantities, as may be seen by referring to (20) and (17). We will be interested only in $u \geq 0$ in the following; then the $n = 0$ term in (23) is, by use of (20),

$$(1 - B)^N Q_o(au) = 0 \quad \text{for } u \geq 0. \tag{24}$$

DETECTION AND FALSE ALARM PROBABILITIES

We now utilize (8), (23), (24), and (20) to obtain the detection probability as

$$P_D = \sum_{n=1}^{N} \binom{N}{n} (1-B)^{N-n} B^n E(a(T - Ln),n-1). \tag{25}$$

Here, B is given by (15), and a is given by (12).

The false alarm probability is obtained by setting $R = 0$, that is, $a = 1$:

$$P_F = \sum_{n=1}^{N} \binom{N}{n} (1-F)^{N-n} F^n E(T - Ln,n-1) . \tag{26}$$

Here, we have utilized (3) et seq., (13), and the fact that B in (15) reduces, for $a = 1$, to

$$\exp(-L) = Q_x(L; \text{noise-only}) = F, \tag{27}$$

which is the fraction of data passed by the nonlinearity in figure 1, for noise-only. In fact, (27) allows us to solve explicitly for the required breakpoint value L, for this exponential example, as

$$L = -\ln(F). \tag{28}$$

To summarize, (25) and (26) give the detection and false alarm probabilities in terms of fundamental quantities

N, number of channels,

F, fraction of data passed,

R, signal-to-noise power ratio per channel,

T, output threshold. (29)

The remaining variables in (25) and (26) are given by (28), (12), and (15) as

$$L = -\ln(F), \quad a = \frac{1}{1+R}, \quad B = \exp(-aL), \tag{30}$$

in order.

SPECIAL CASES

For N = 1, one channel, (25) and (26) reduce to

$$P_F = F\,Q_1(T-L) = \begin{cases} F & \text{for } 0 \le T < L \\ \exp(-T) & \text{for } L \le T \end{cases},$$

$$P_D = B\,Q_1(a(T-L)) = \begin{cases} F^a & \text{for } 0 \le T < L \\ \exp(-aT) & \text{for } L \le T \end{cases}. \tag{31}$$

That is, $P_D = P_F{}^a$ for N = 1, independent of the value of fraction F. This is obvious from (13) in this case.

Instead, if fraction F = 1, that is, no nonlinearity, then (28) and (15) yield L = 0, B = 1, and we find

$$\left.\begin{aligned} P_F &= E(T,N-1) \\ P_D &= E(aT,N-1) \end{aligned}\right\} \quad \text{for } F = 1. \tag{32}$$

These results agree with [5; (7) and (8)]. A program for the evaluation of general results (25) and (26), as well as the special case (32), is presented in the appendix.

GRAPHICAL RESULTS

In figure 3*, the receiver operating characteristic (ROC) is given for $N = 1$, $F = 1$. That is, there is one channel and the nonlinearity is not active. There is no need to consider values of F less than 1, according to the comment under (31); however, see the subsection below on achievable false alarm values. The curves in figure 3 are parameterized according to

$$R(\mathrm{dB}) = 10 \log R. \tag{33}$$

The remaining fundamental quantity, threshold T in (29), has been eliminated, and P_D is plotted versus P_F on normal probability paper.

In figures 4, 5, 6, 7, the number of channels is kept at $N = 2$, while fraction

$$F = 1, .1, .01, .001, \tag{34}$$

respectively. Additional cases for

$$N = 4, 6, 8, 16, 32, 64, \tag{35}$$

in figures 8 through 31, complete the coverage in a similar fashion.

---

*Figures 3 through 31 are grouped at the end of this section.

ACHIEVABLE FALSE ALARM VALUES

Not all values of false alarm probability can be reached by the processing system of figure 1. Since the nonlinear device output y_n can only take on the values $y_n = 0$ and $y_n \geq L$, the sum z can only assume the values $z = 0$ and $z \geq L$. Also, since the probability of $z = 0$ is $(1 - F)^N$ for noise-only, where F is the fraction of data passed by the nonlinearity in each channel, then the probability of getting $z \geq L$ is $1 - (1 - F)^N$. Thus, the range of reachable false alarm probabilities is

$$P_F \leq 1 - (1 - F)^N. \tag{36}$$

This bound holds regardless of the form of the probability density for random variables $\{x_n\}$ in figure 1.

For the special case of $N = 1$, this rule yields $P_F \leq F$. Thus, the plot in figure 3 for $N = 1$, $F = 1$ must be modified for $F < 1$, to the extent that only the values for $P_F \leq F$ are achievable.

For $N > 1$, the rule in (36) first becomes obvious in figure 6 for $N = 2$, $F = .01$. Namely, (36) yields

$$P_F \leq 1 - (1 - .01)^2 = .0199. \tag{37}$$

Thus, the curves in figure 6 are terminated to the right of this value of the false alarm probability. This termination feature occurs in numerous other figures, always governed by (36).

ERRATIC BEHAVIOR OF RECEIVER OPERATING CHARACTERISTICS

Some of the curves develop significant kinks for larger values of the false alarm probability; see figure 31 for the most pronounced example in this set of results. This behavior is not due to computer round-off error; rather, it is due to the shifted components of probability density function (22) "kicking in" when the output threshold reaches various multiples of breakpoint L. Equivalently, the shifted E-function components of the detection probability and false alarm probability in (25) and (26) are activated at different threshold levels, reflecting the inherent abrupt change of behavior of these functions at zero argument. For example,

$$E(u,0) = \left\{ \begin{array}{ll} 1 & \text{for } u < 0 \\ \exp(-u) & \text{for } u \geq 0 \end{array} \right\},$$

$$E(u,1) = \left\{ \begin{array}{ll} 1 & \text{for } u < 0 \\ \exp(-u)(1+u) & \text{for } u \geq 0 \end{array} \right\}. \tag{38}$$

Thus, E(u,0) has a discontinuous slope at u = 0, while E(u,1) has a discontinuous second derivative at u = 0.

OBSERVATIONS

The required signal-to-noise ratios for various values of N and F are presented in tables 1 and 2 for two different levels of performance, as read directly from figures 3 through 31. The overriding impression is that the degradation in performance is not severe, even for small values of F, the fraction of data passed by the nonlinearity. For example, from table 1, the decibel difference at F = .001 versus F = 1 is, for N = 1, 2, 4, 6, 8, 16, 32, 64, respectively, just

0, 0.3, 0.8, 1.0, 1.2, 1.7, 2.2, 2.7 dB.

For table 2, these differences are substantially the same:

0, 0.2, 0.5, 0.8, 1.0, 1.5, 2.0, 2.5 dB.

Thus, the losses increase from 0 dB at N = 1 channel, to less than 3 dB for N = 64 channels.

The situation is slightly worse for the lower-quality case of P_F = 1E-3, P_D = .5. Namely, as F is changed from 1 to .001, the required increment in signal-to-noise ratio is 0 dB for N = 1, whereas it is 3.4 dB for N = 64.

N	Required R(dB) for F = 1	.1	.01	.001
1	12.8	12.8	12.8	12.8
2	9.5	9.6	9.7	9.8
4	6.8	6.9	7.2	7.6
6	5.4	5.5	5.9	6.4
8	4.5	4.6	5.1	5.7
16	2.3	2.6	3.2	4.0
32	0.4	0.7	1.6	2.6
64	-1.4	-1.0	0.1	1.3

Table 1. Required Signal-to-Noise Ratio for P_F = 1E-6, P_D = .5

N	Required R(dB) for F = 1	.1	.01	.001
1	22.4	22.4	22.4	22.4
2	16.0	16.0	16.1	16.2
4	11.6	11.6	11.8	12.1
6	9.4	9.6	9.9	10.2
8	8.1	8.2	8.6	9.1
16	5.3	5.6	6.1	6.8
32	2.9	3.3	4.0	4.9
64	0.8	1.2	2.2	3.3

Table 2. Required Signal-to-Noise Ratio for P_F = 1E-8, P_D = .9

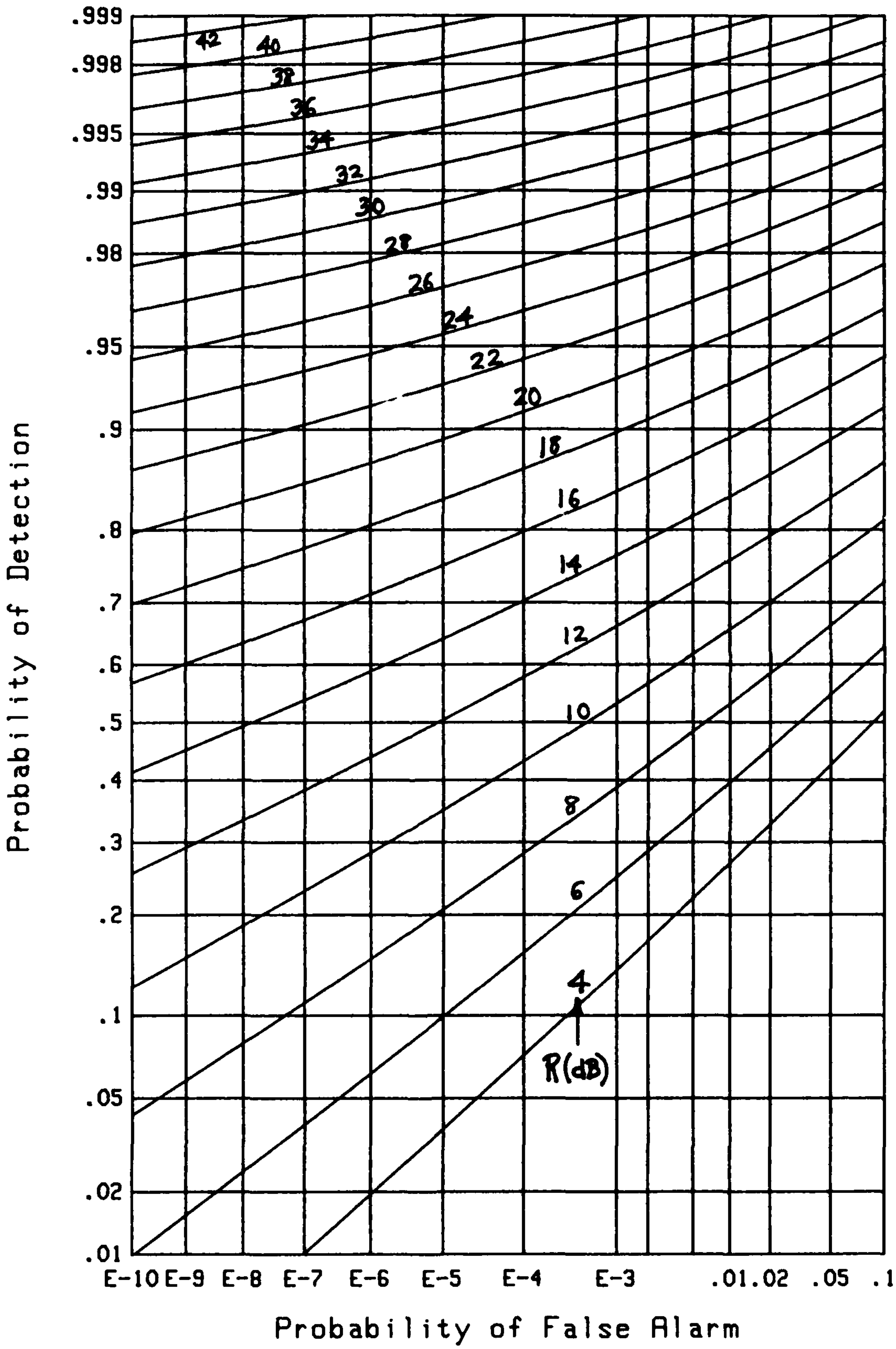

Figure 3. ROC for N=1, F=1.

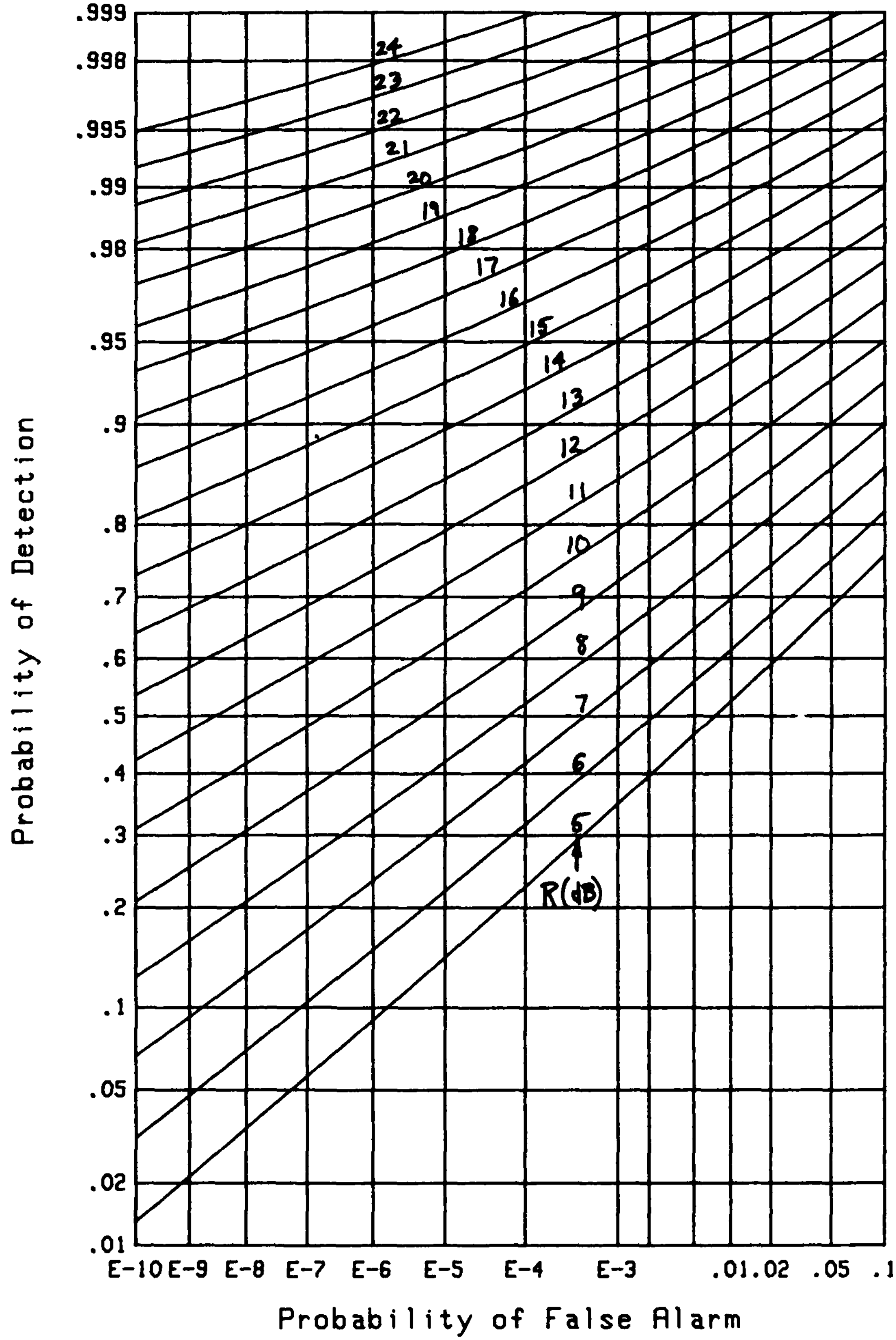

Figure 4. ROC for N=2, F=1.

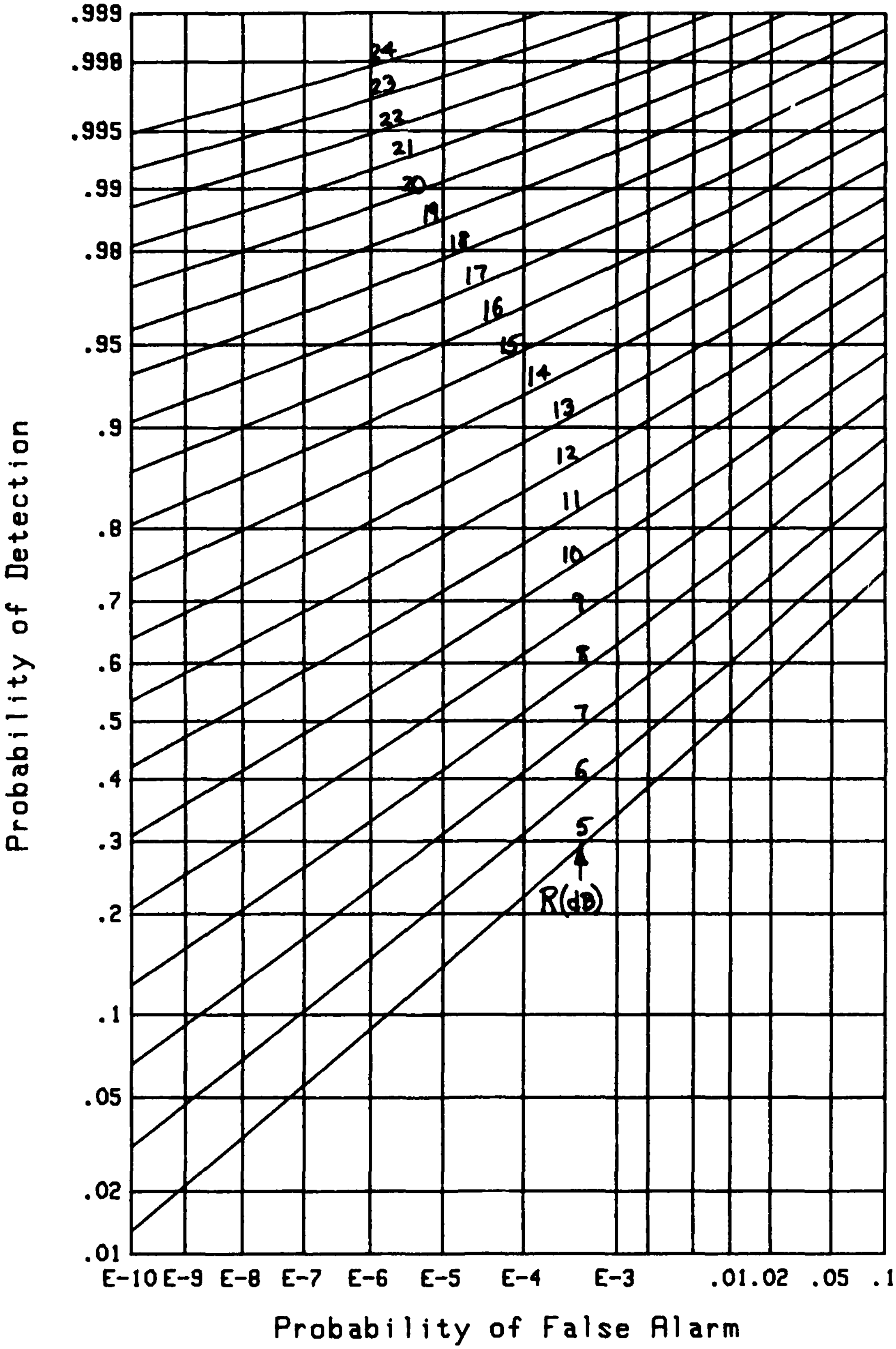

Figure 5. ROC for N=2, F=.1

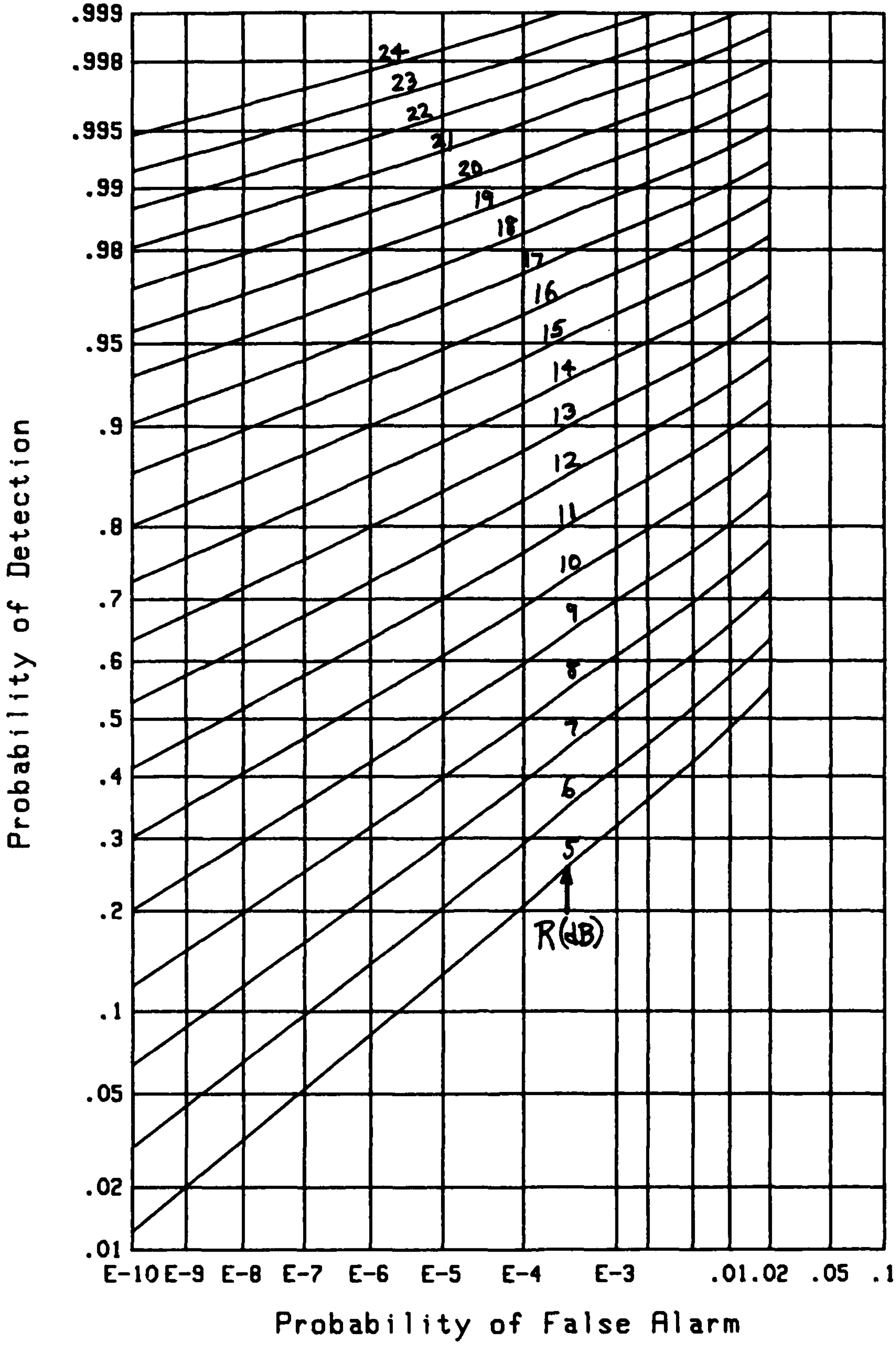

Figure 6. ROC for N=2, F=.01

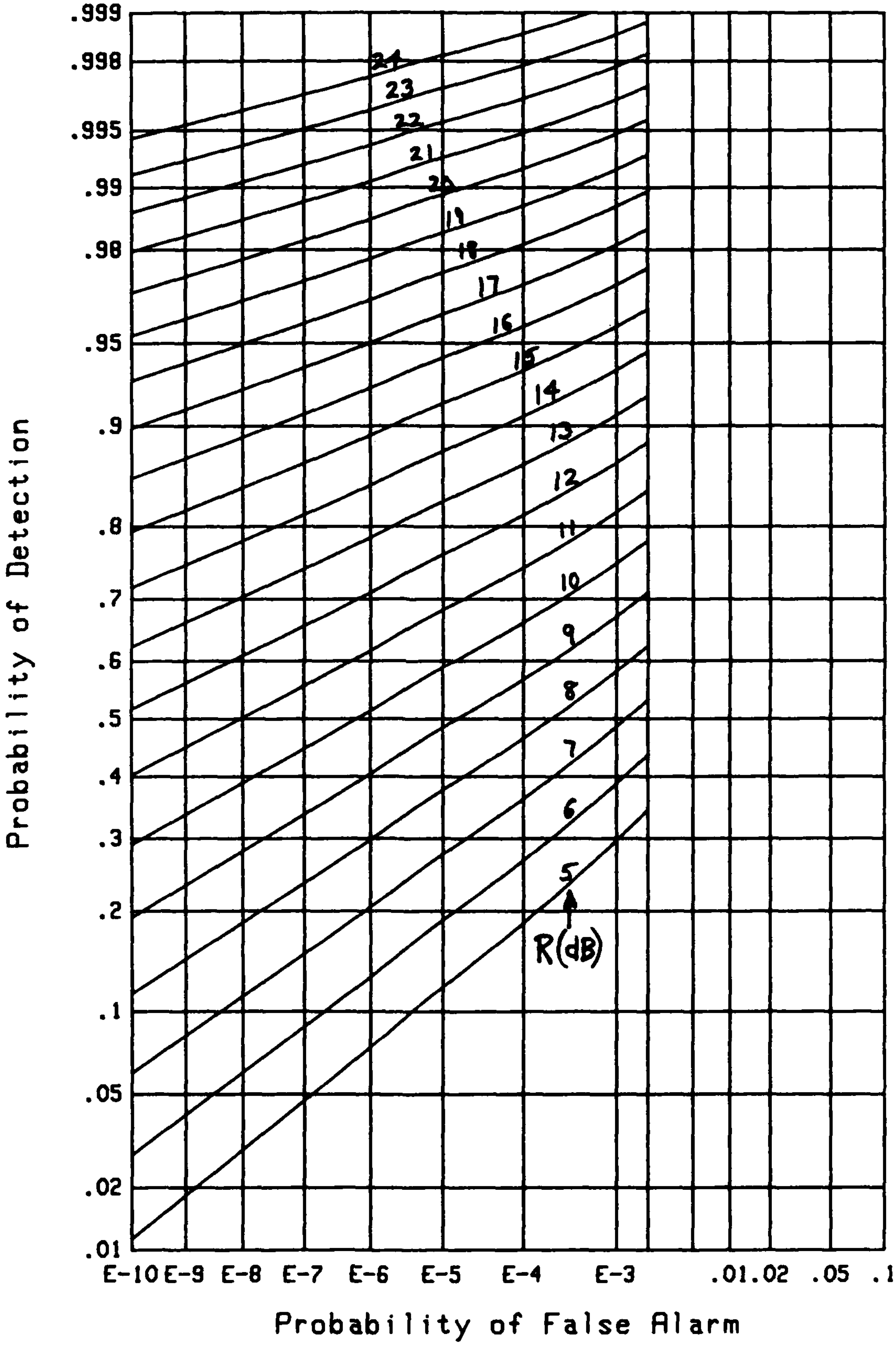

Figure 7. ROC for N=2, F=.001

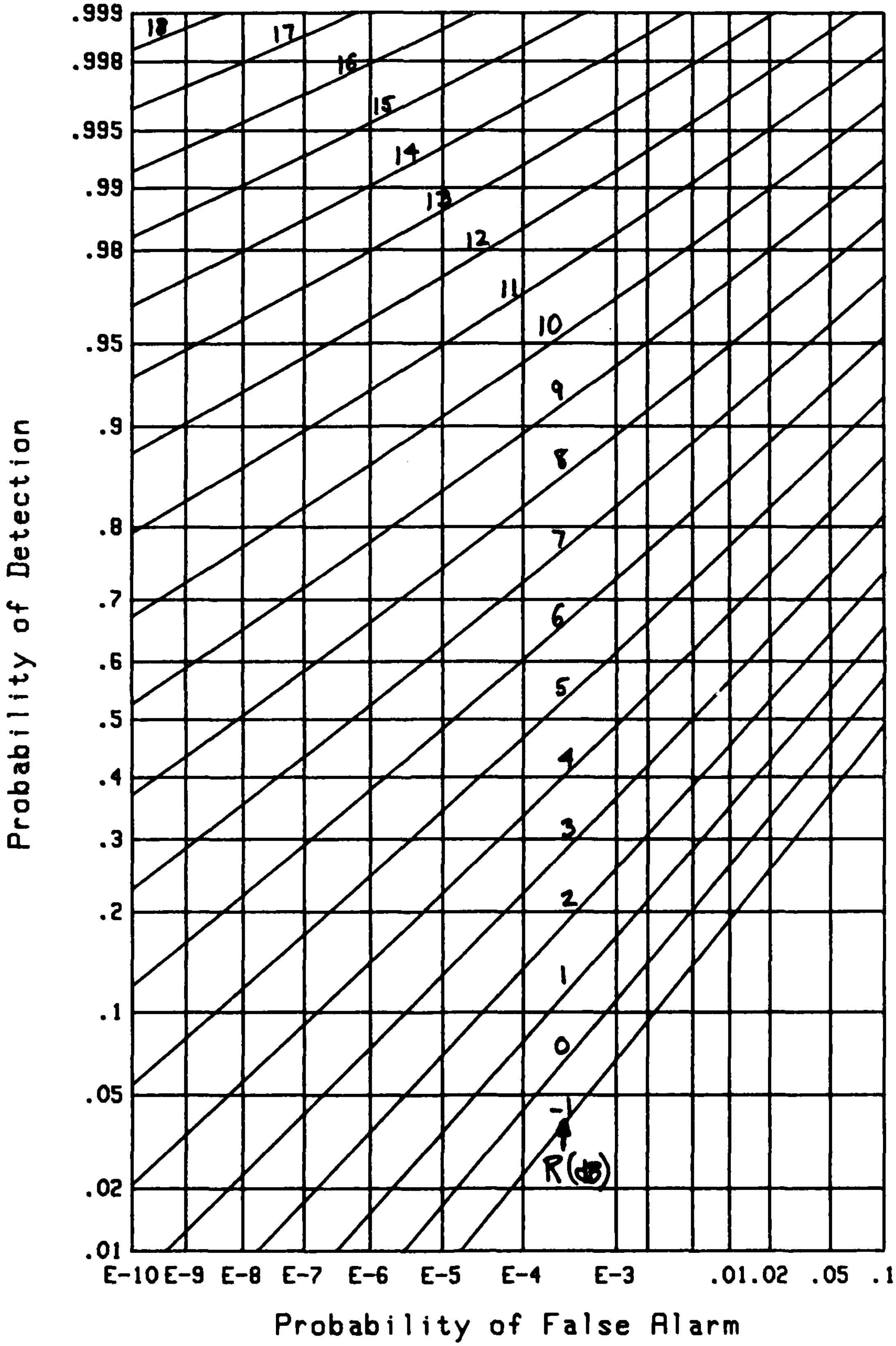

Figure 8. ROC for N=4, F=1.

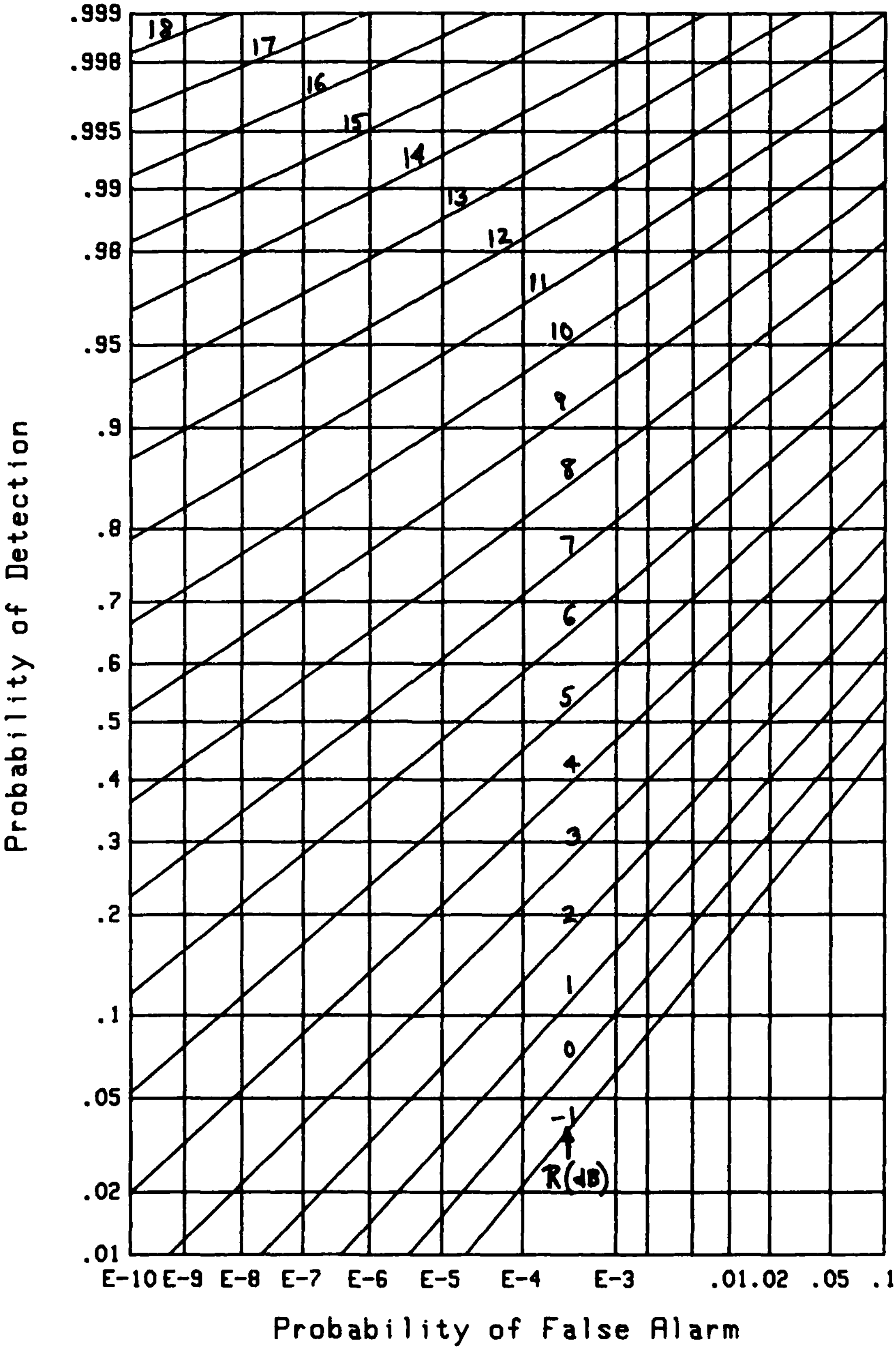

Figure 9. ROC for N=4, F=.1

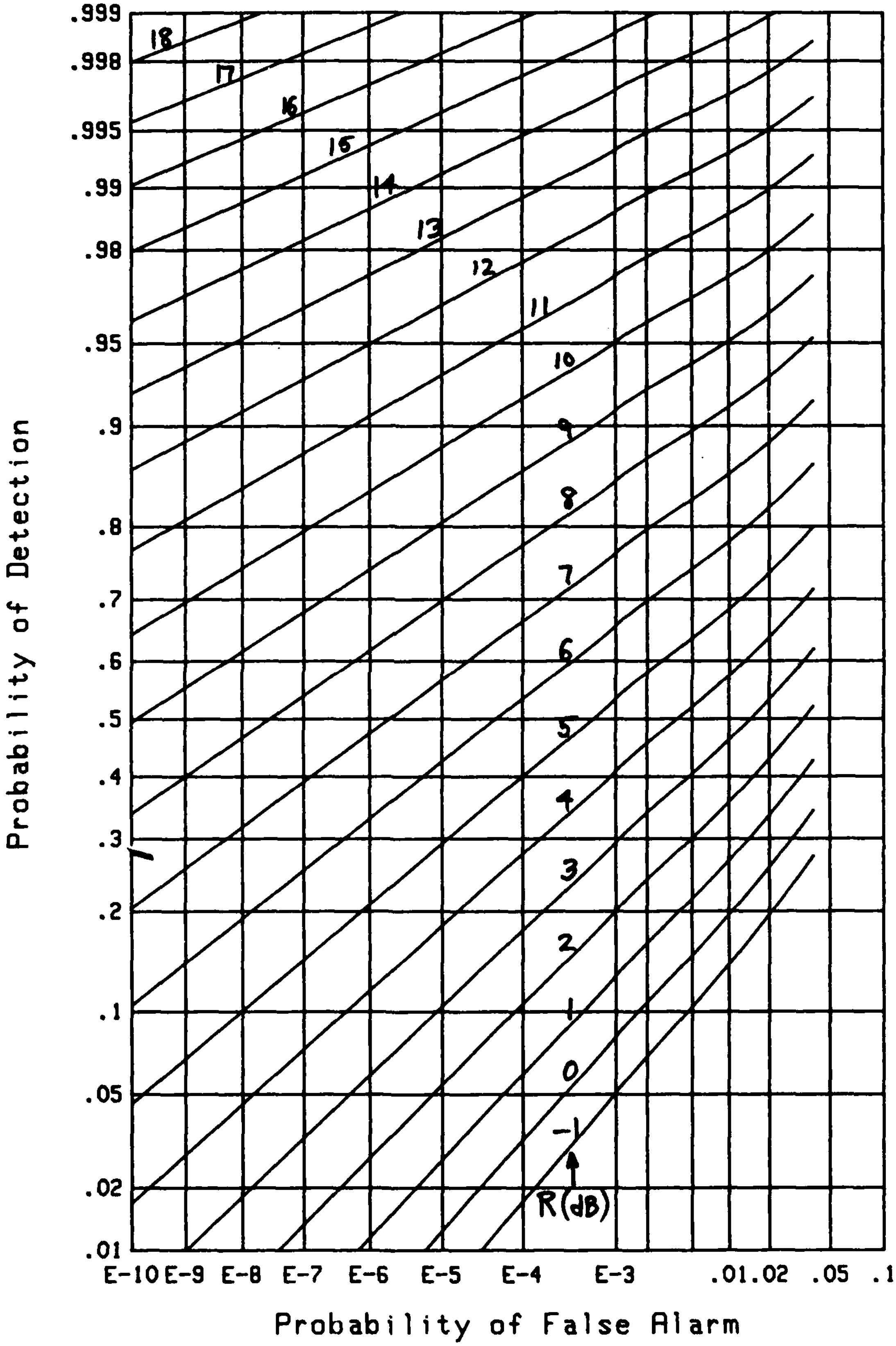

Figure 10. ROC for N=4, F=.01

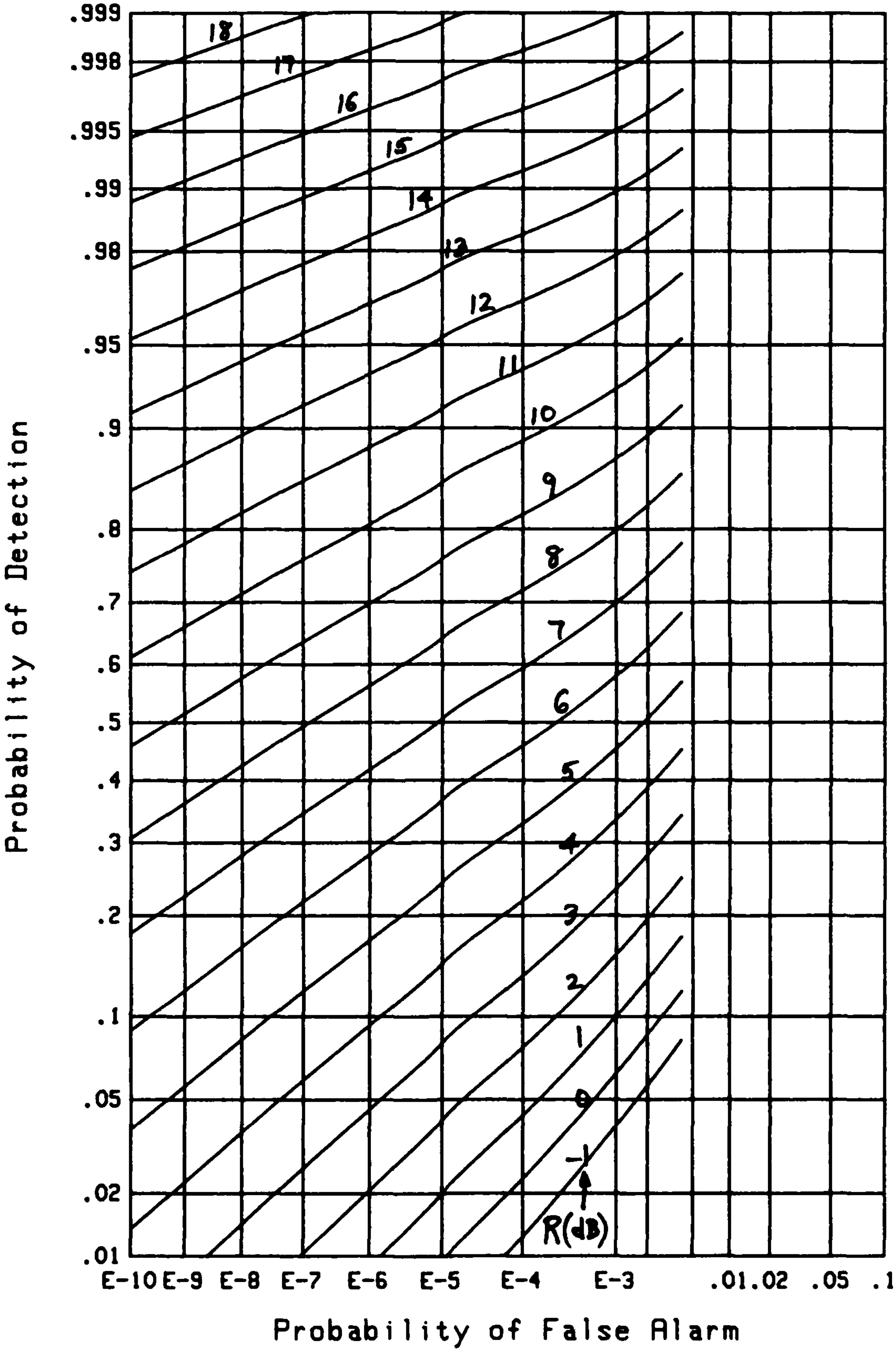

Figure 11. ROC for N=4, F=.001

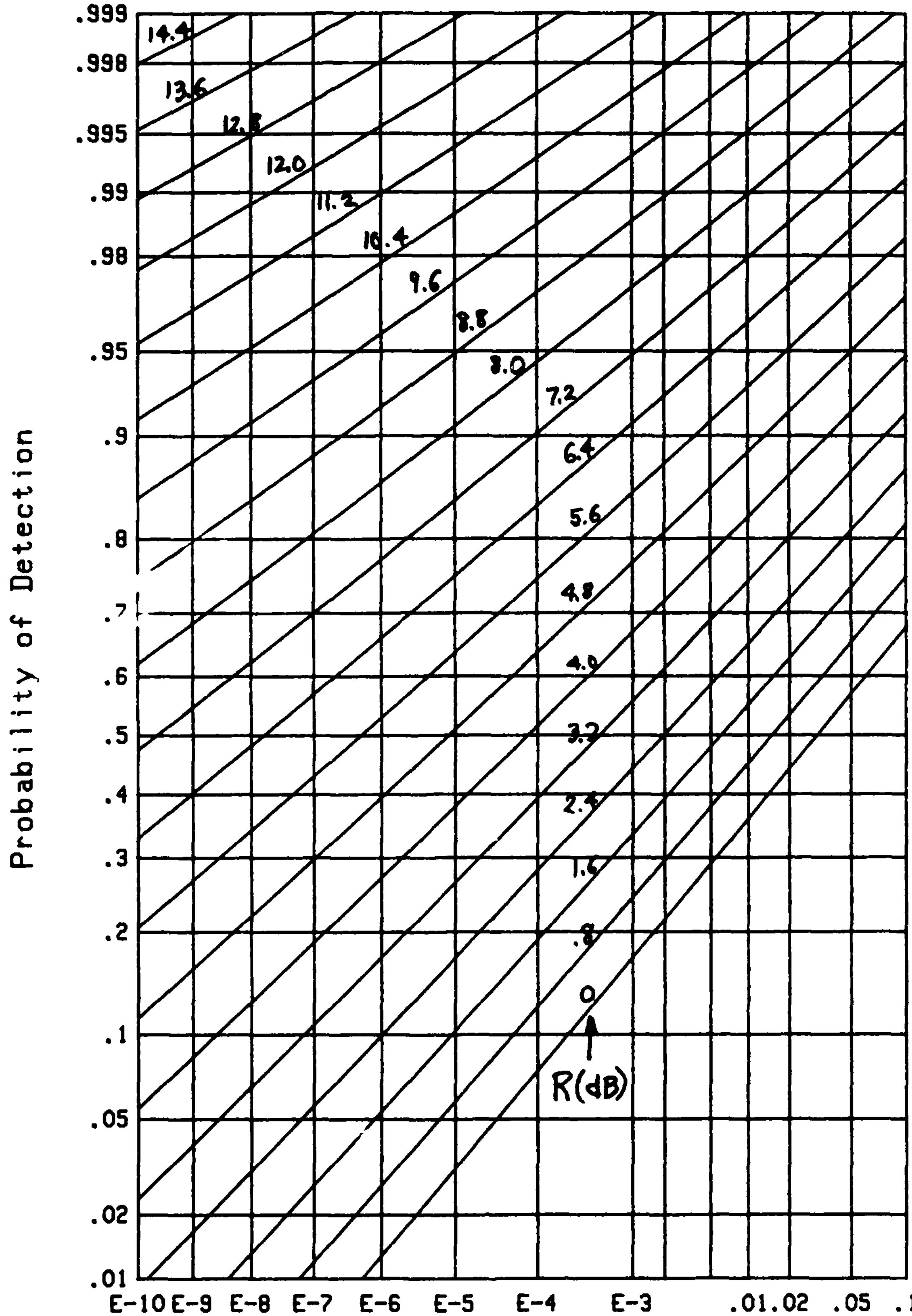

Figure 12. ROC for N=6, F=1.

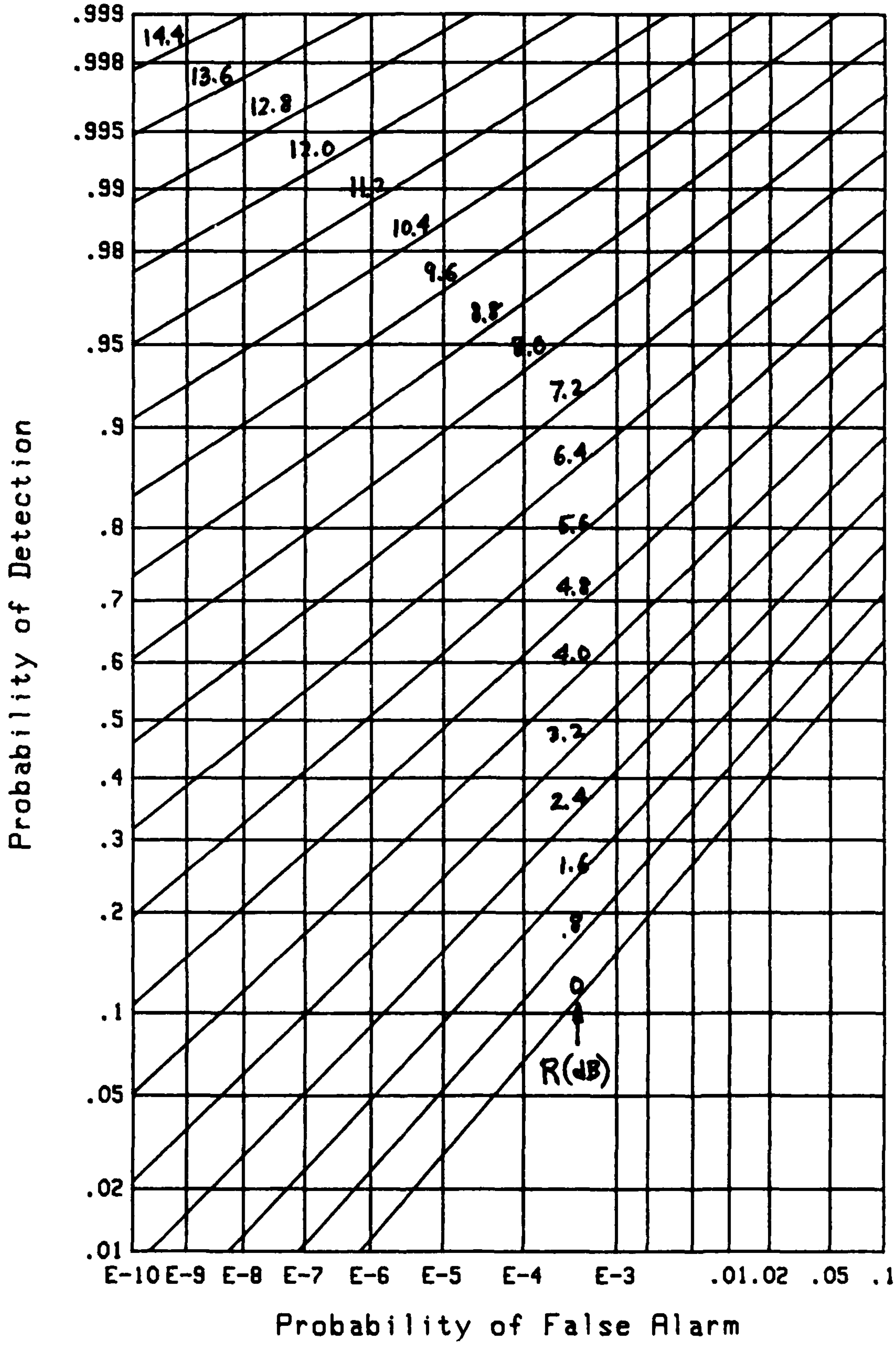

Figure 13. ROC for N=6, F=.1

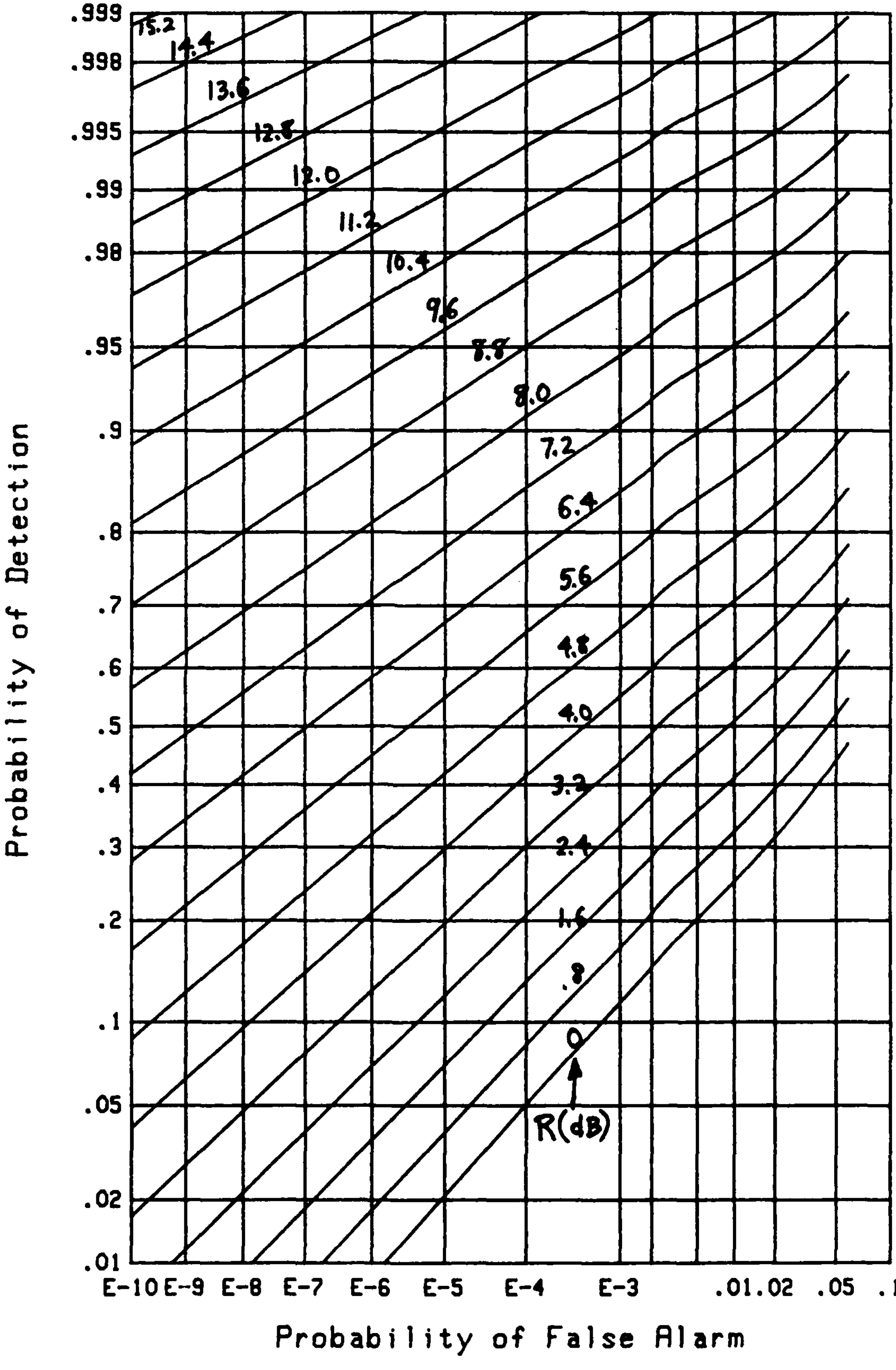

Figure 14. ROC for N=6, F=.01

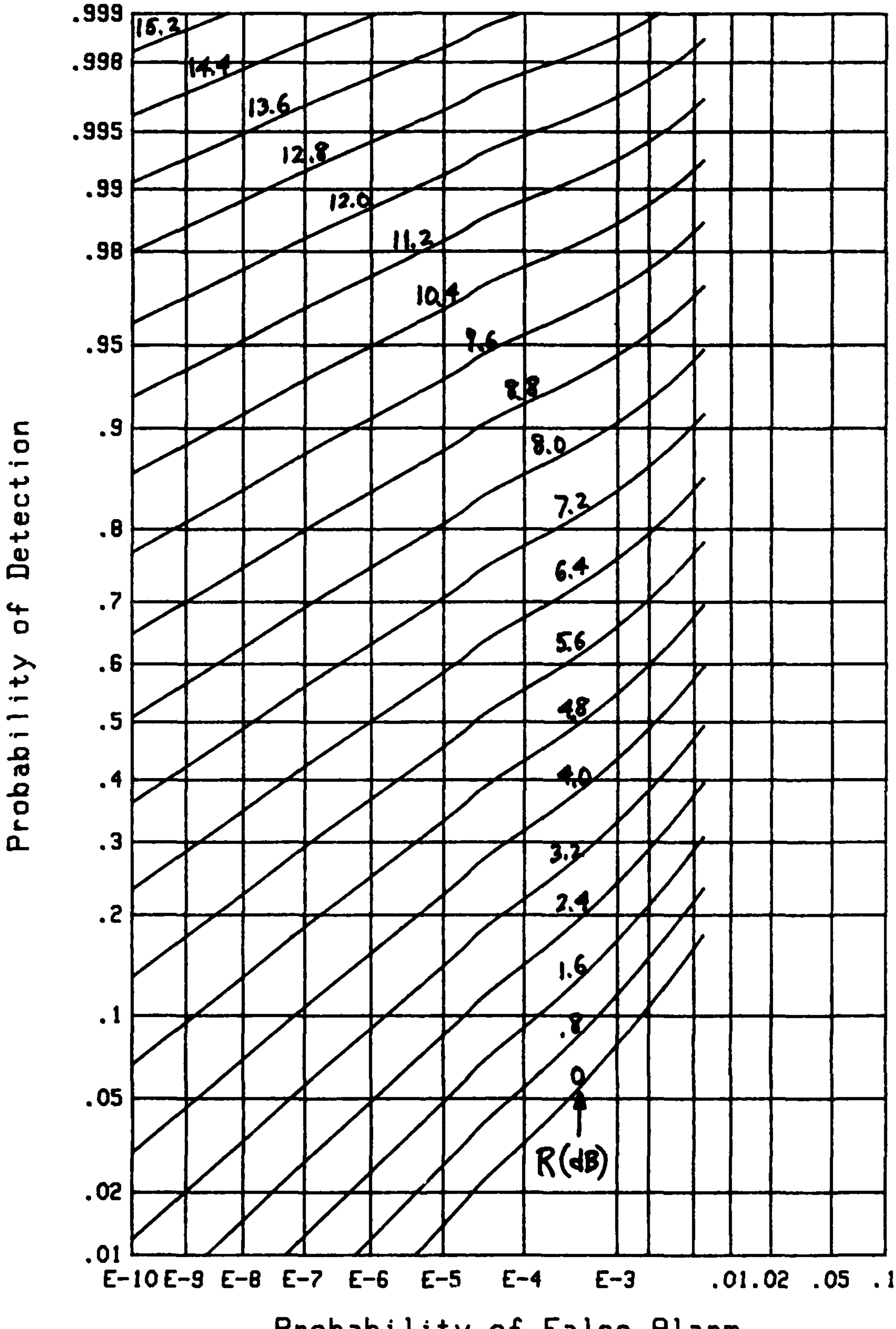

Figure 15. ROC for N=6, F=.001

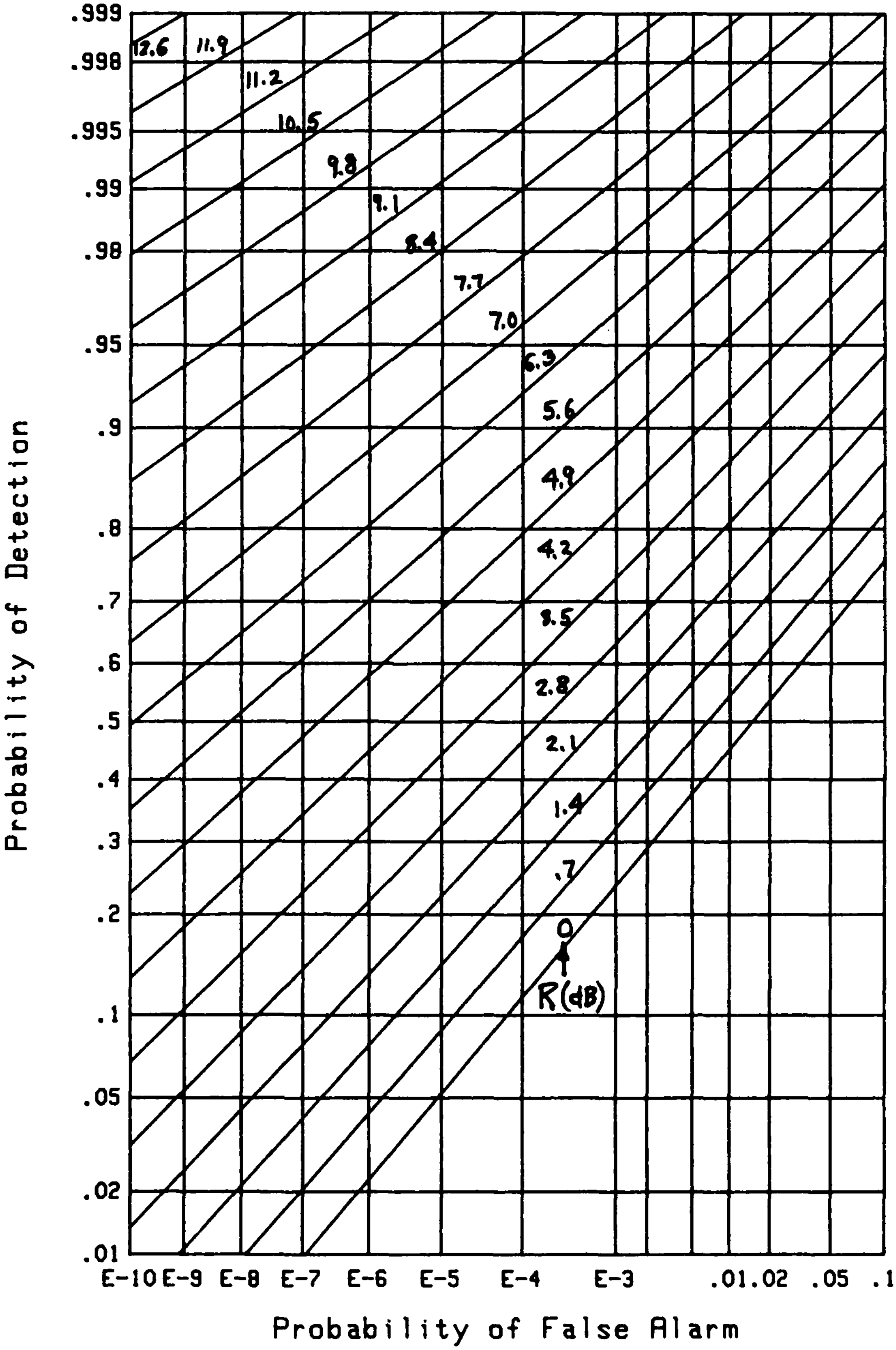

Figure 16. ROC for N=8, F=1.

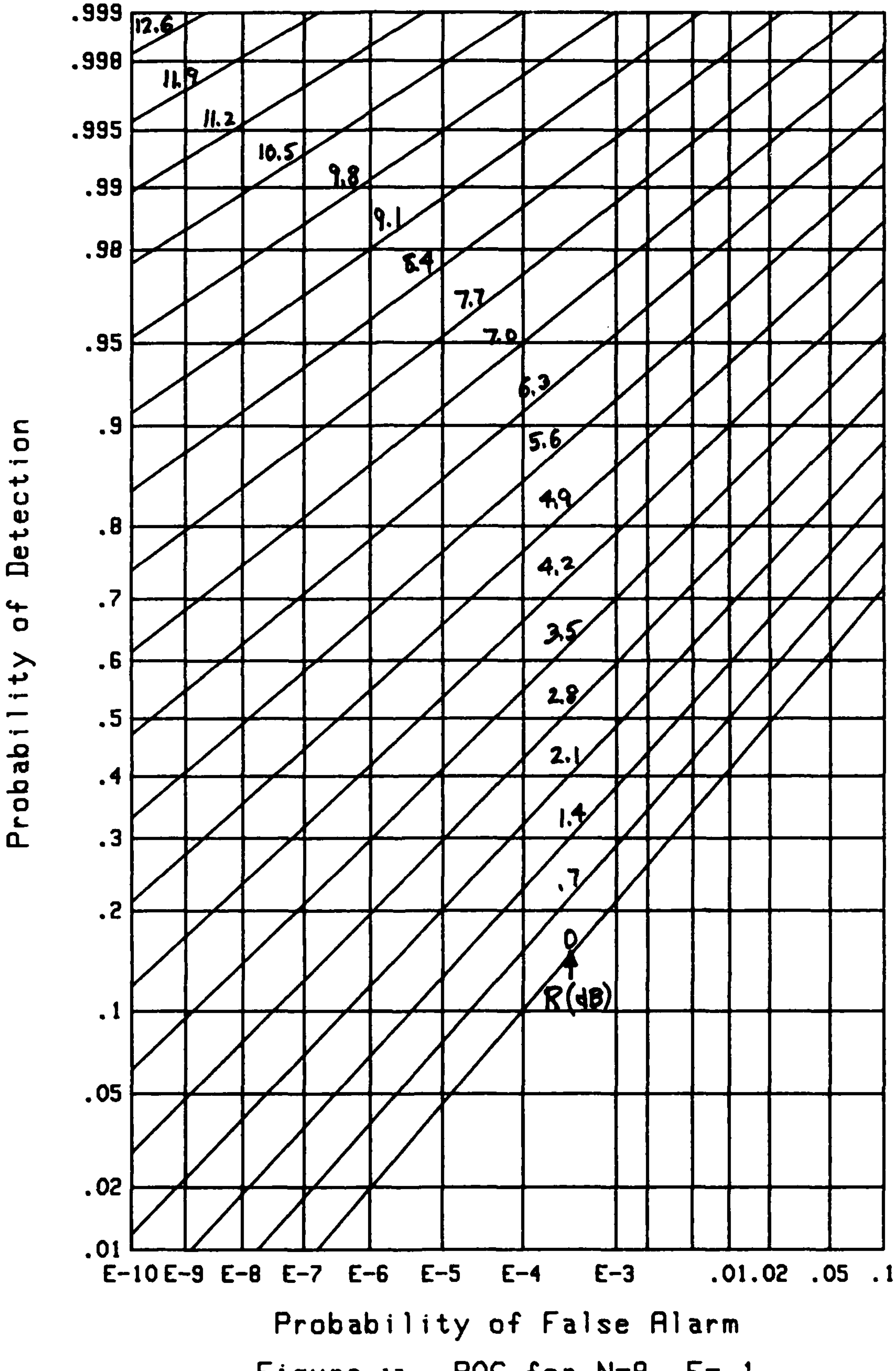

Figure 17. ROC for N=8, F=.1

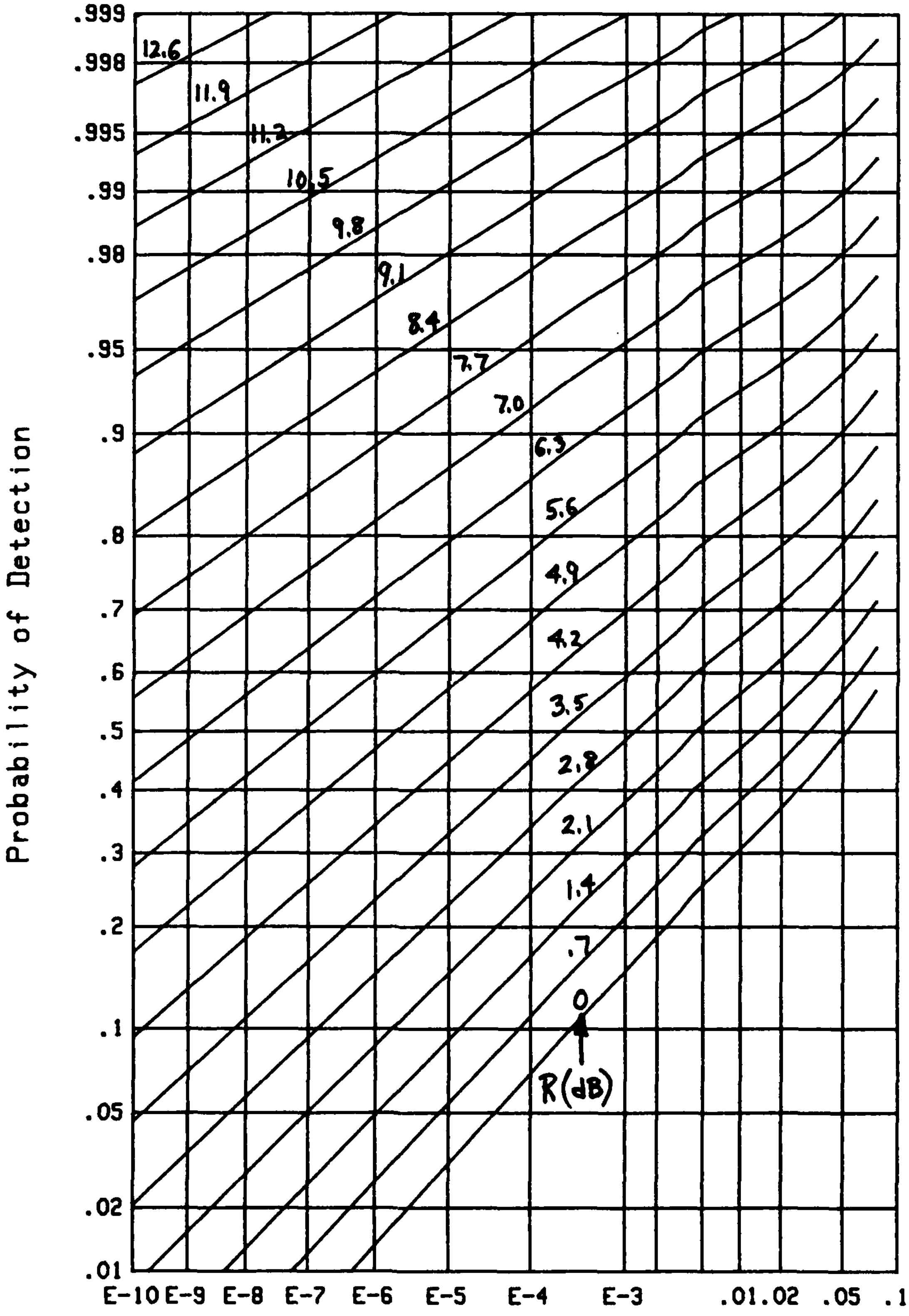

Figure 18. ROC for N=8, F=.01

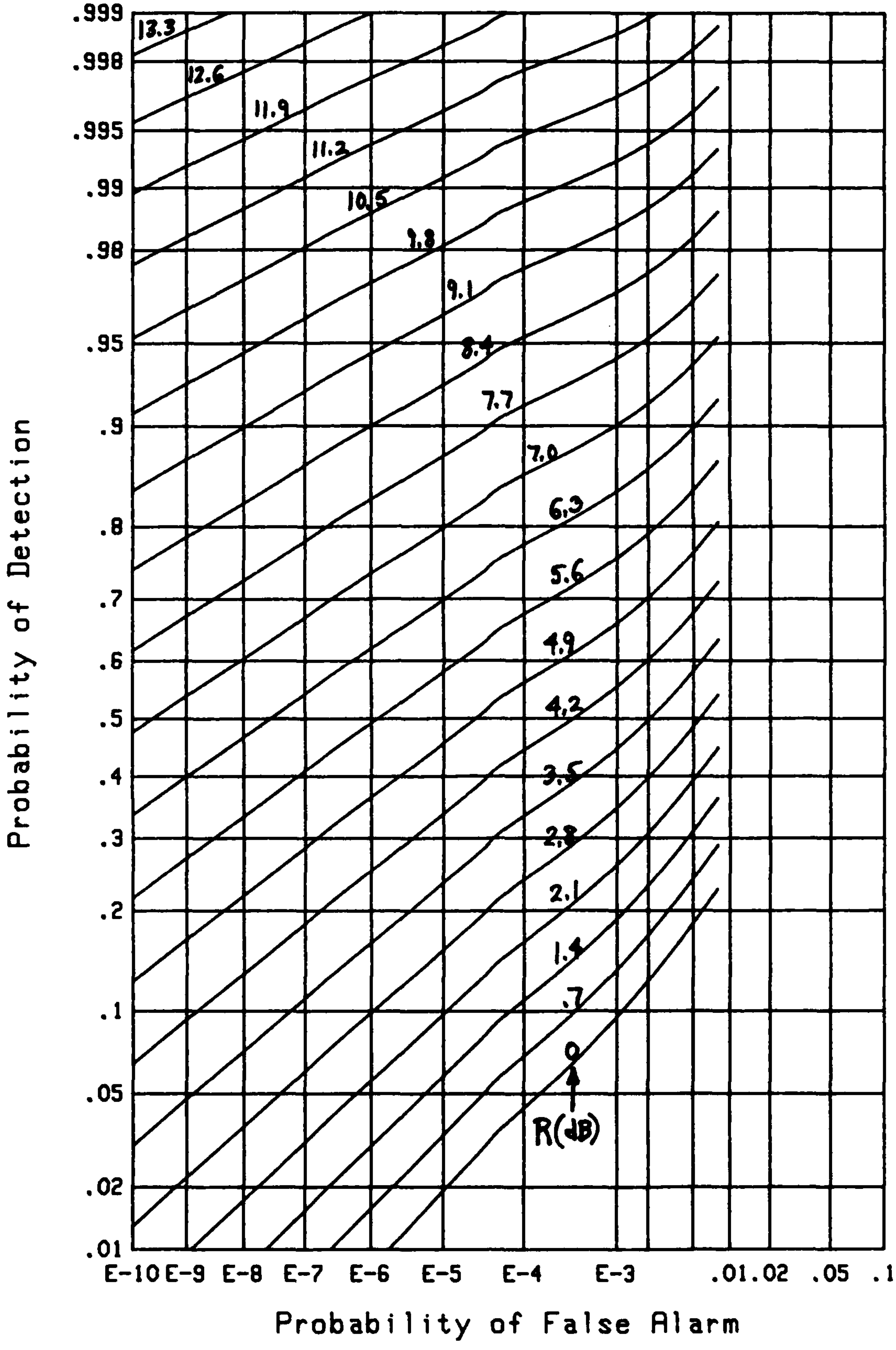

Figure 19. ROC for N=8, F=.001

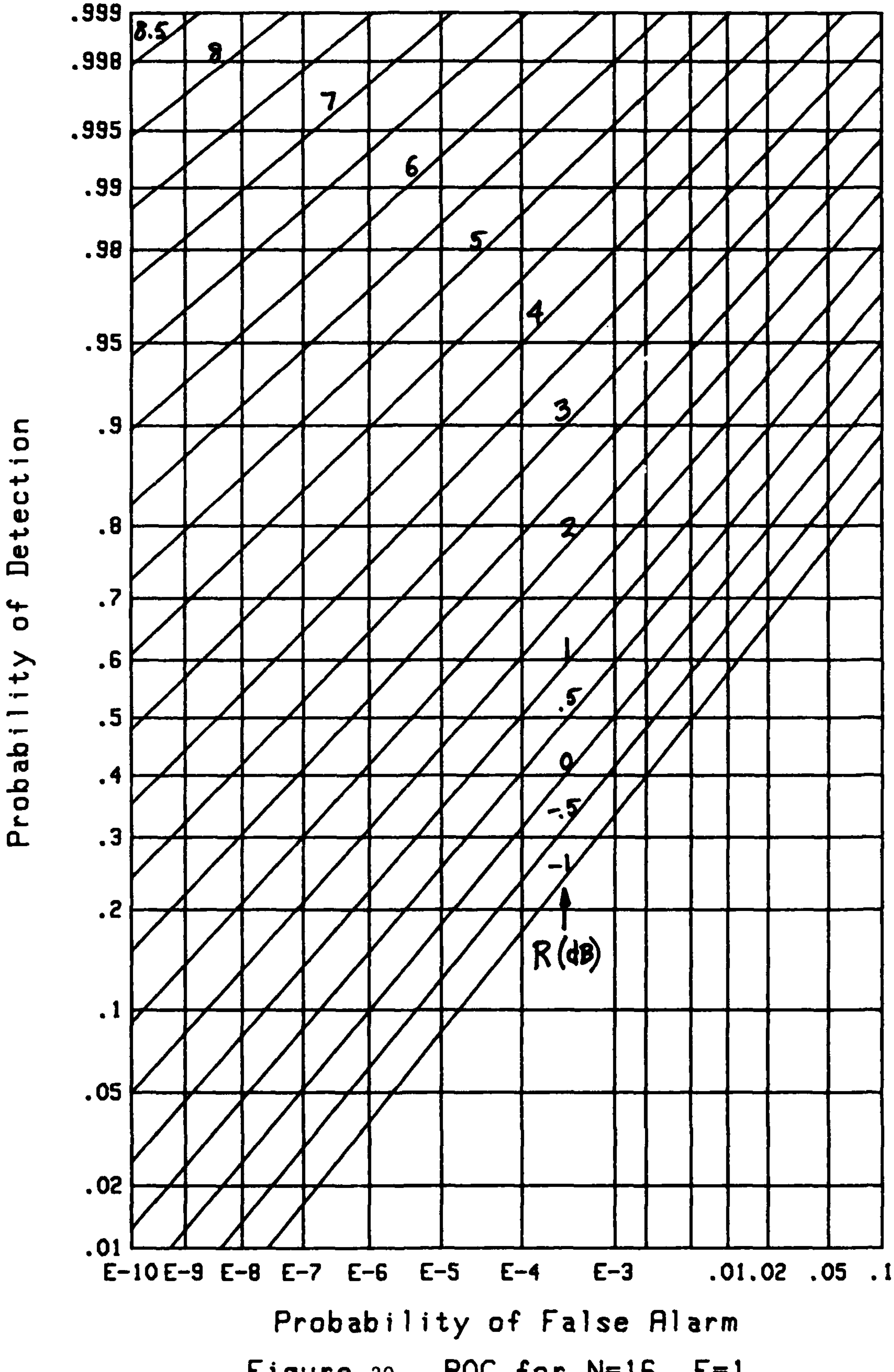

Figure 20. ROC for N=16, F=1.

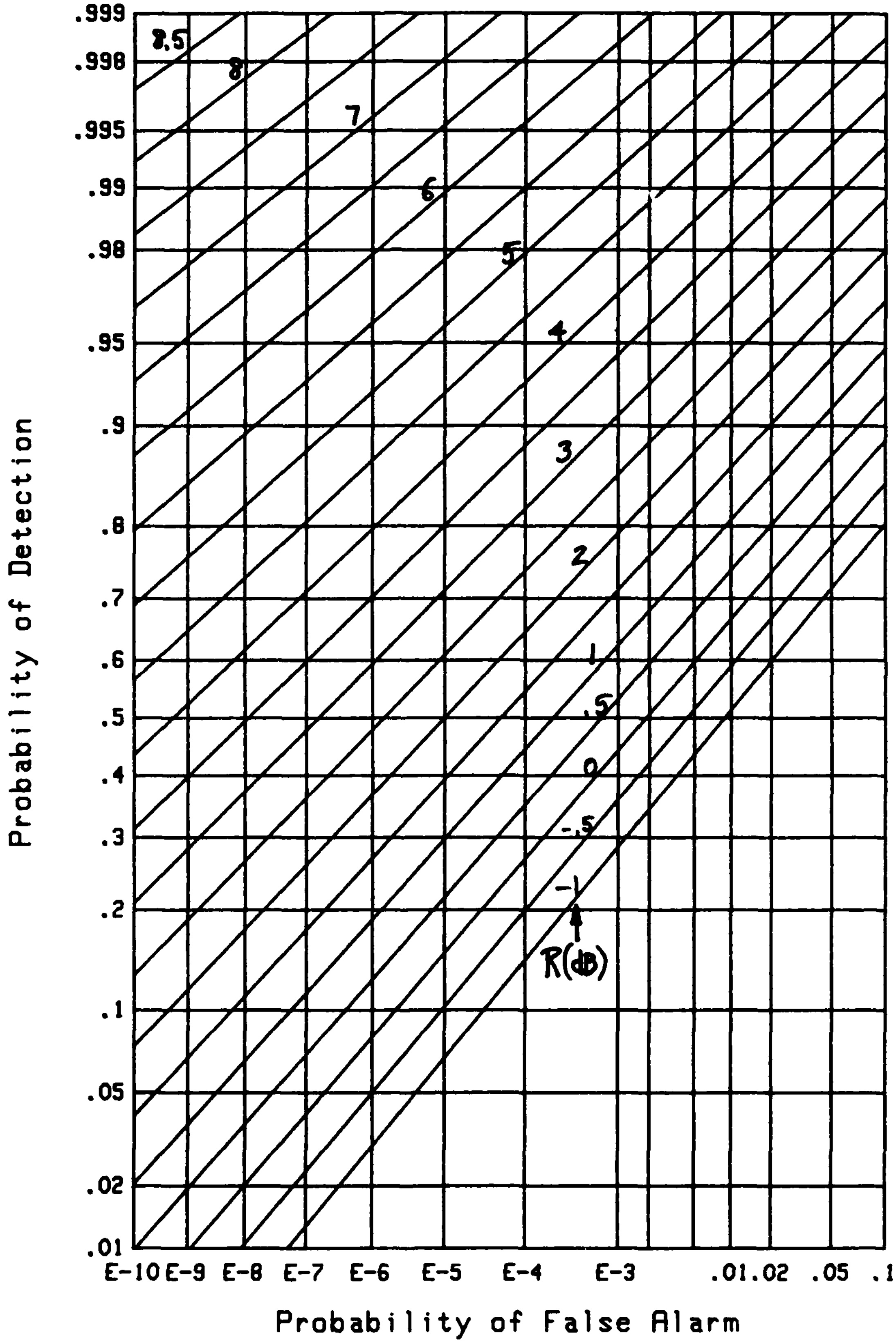

Figure 21. ROC for N=16, F=.1

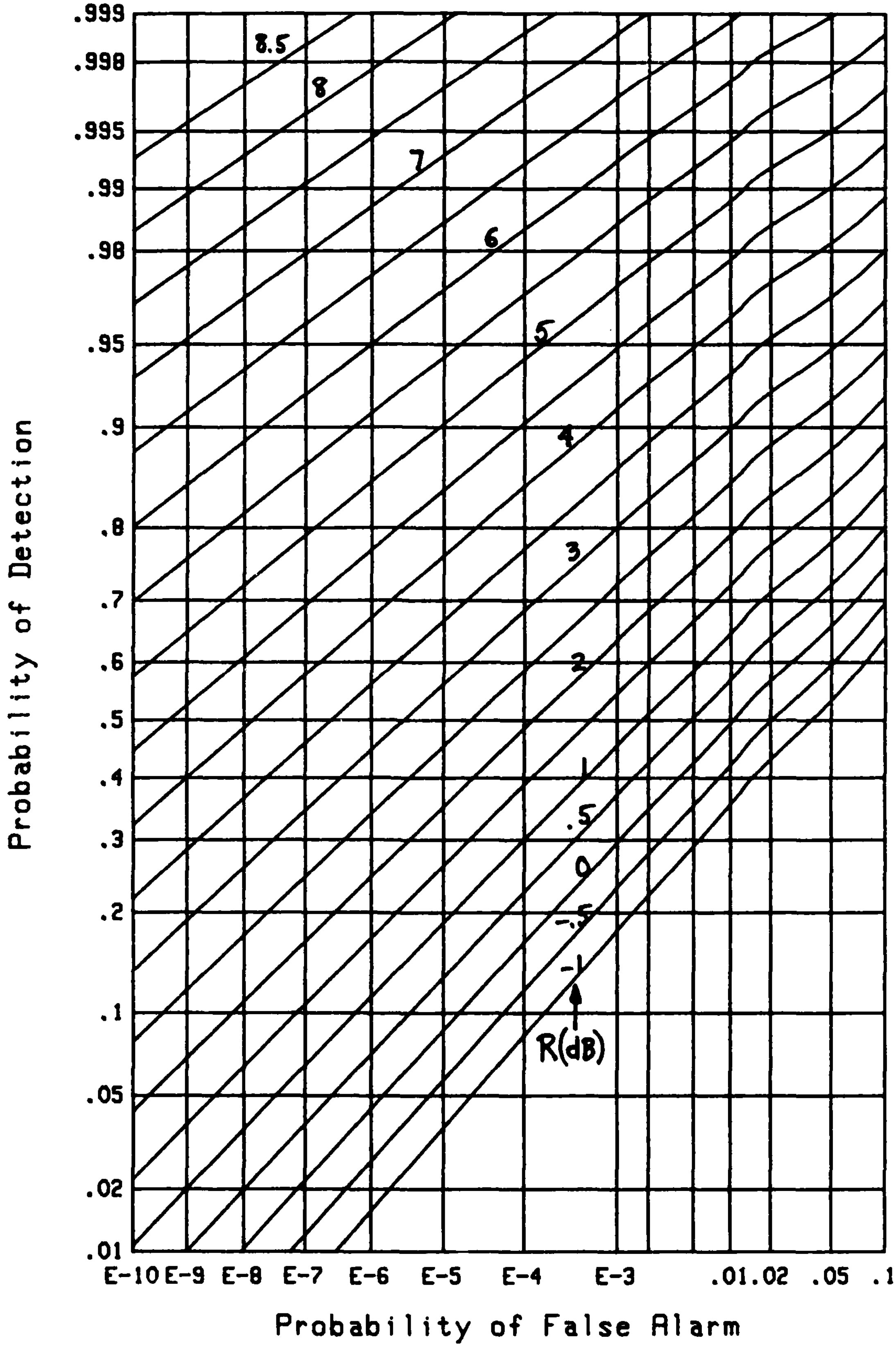

Figure 22. ROC for N=16, F=.01

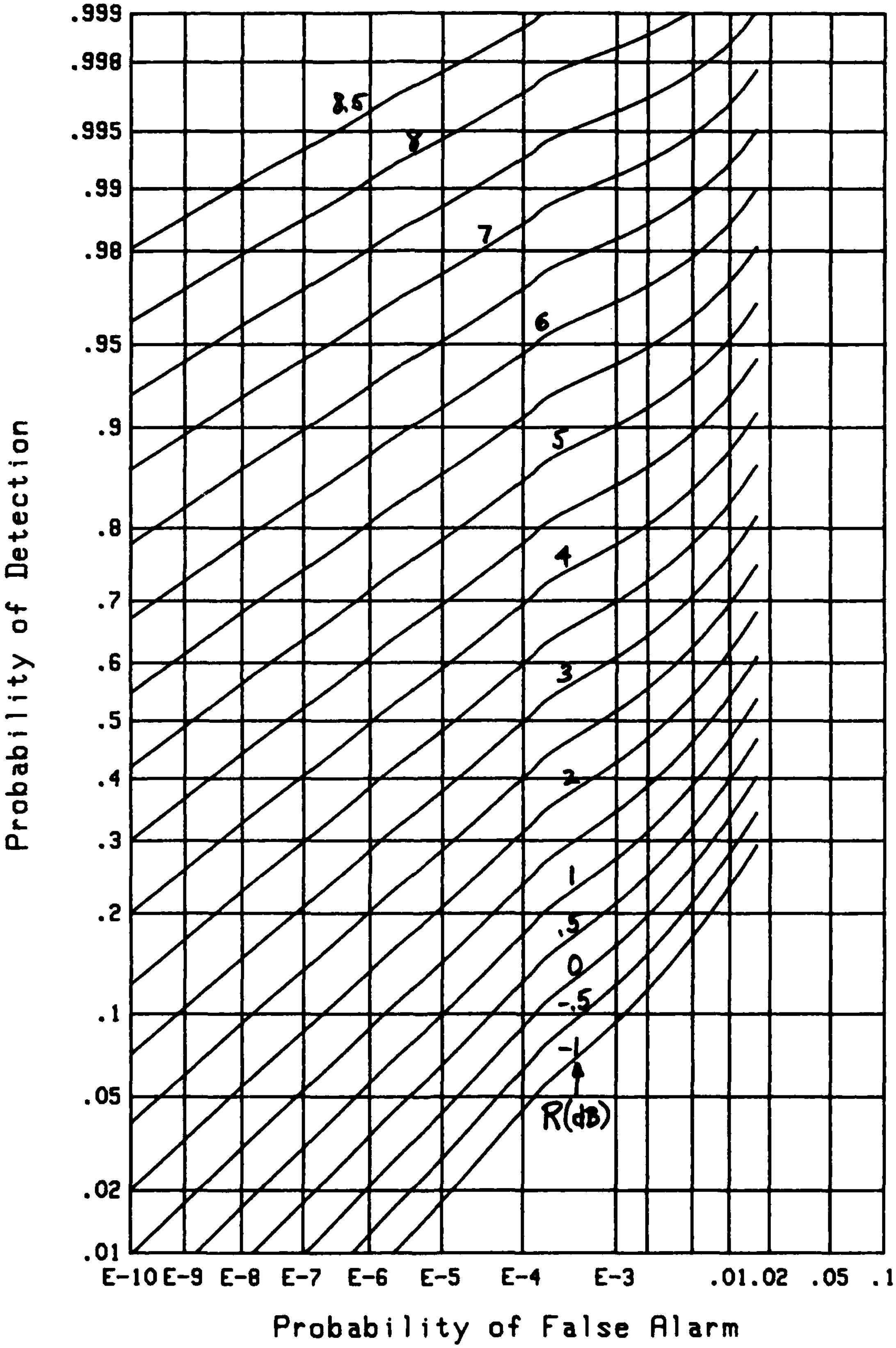

Figure 23. ROC for N=16, F=.001

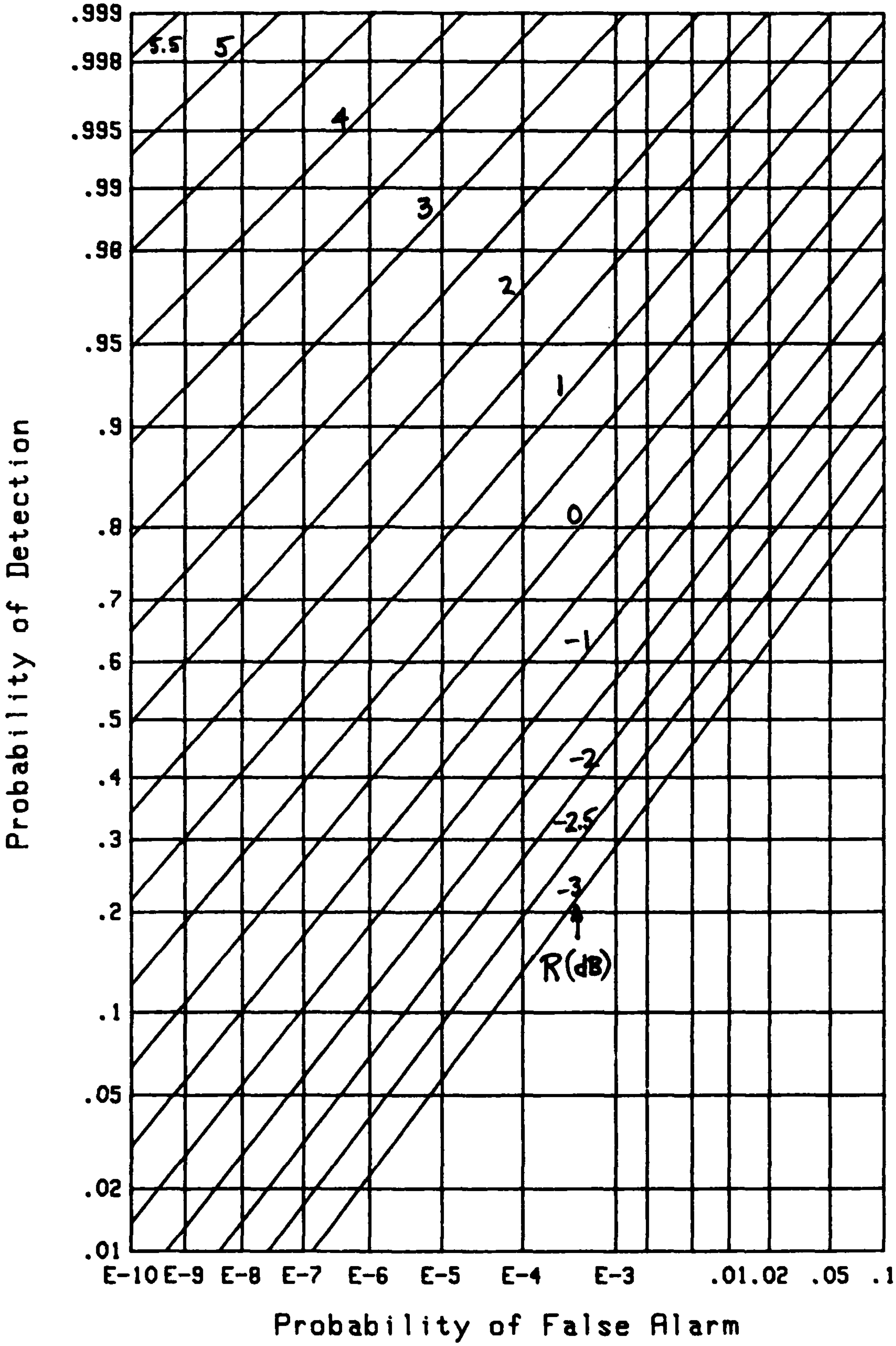

Figure 24. ROC for N=32, F=1.

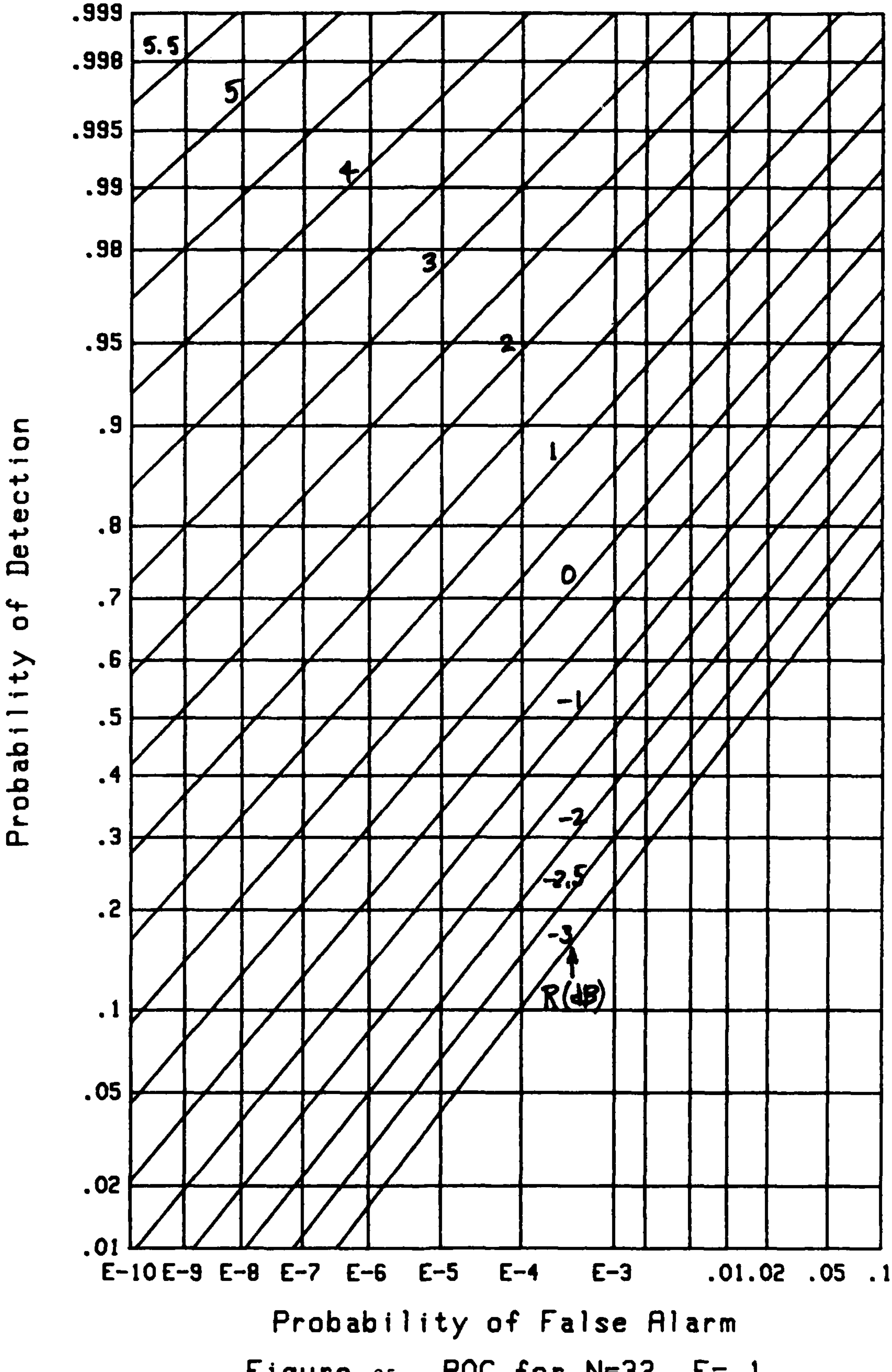

Figure 25. ROC for N=32, F=.1

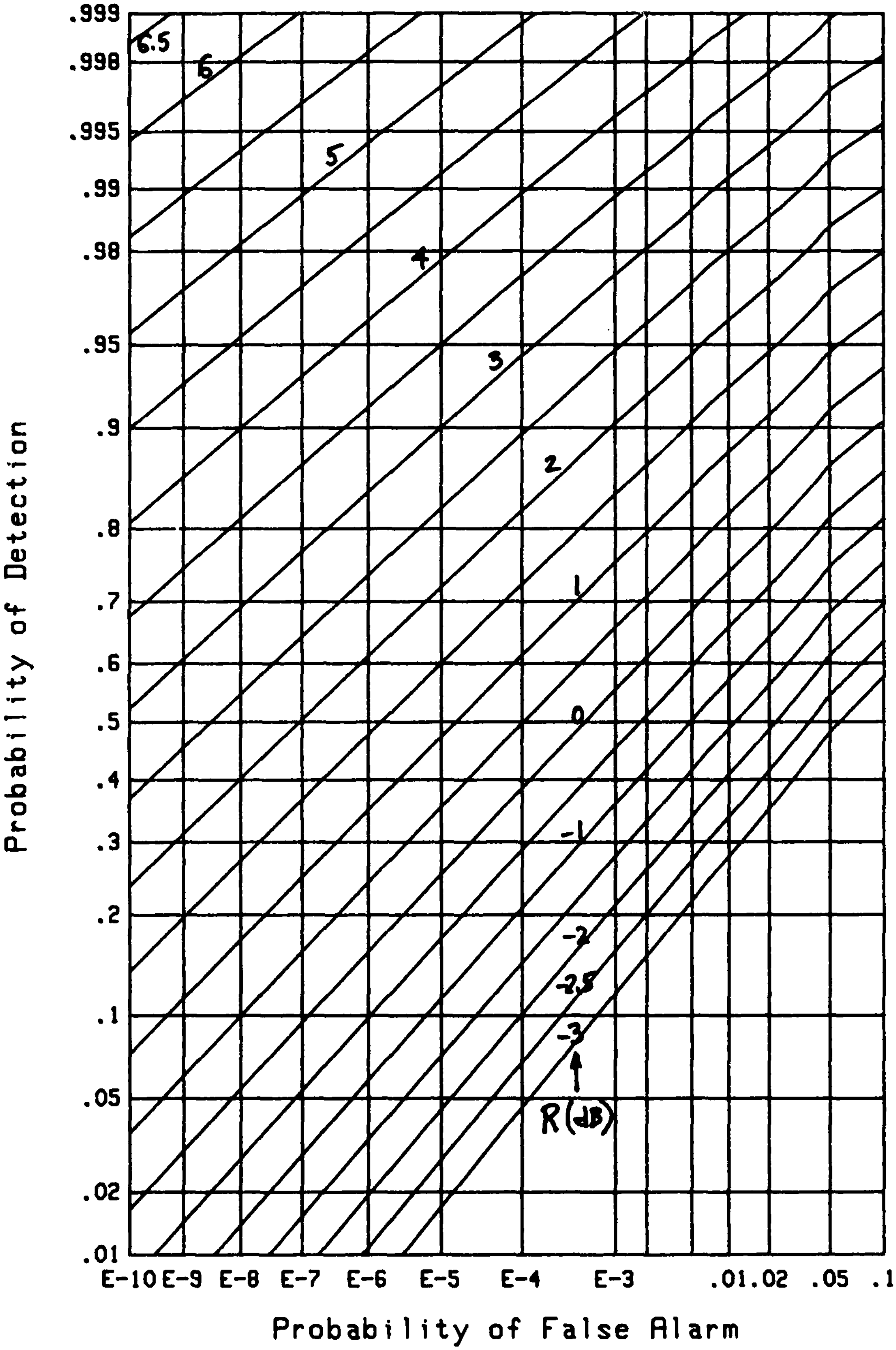

Figure 26. ROC for N=32, F=.01

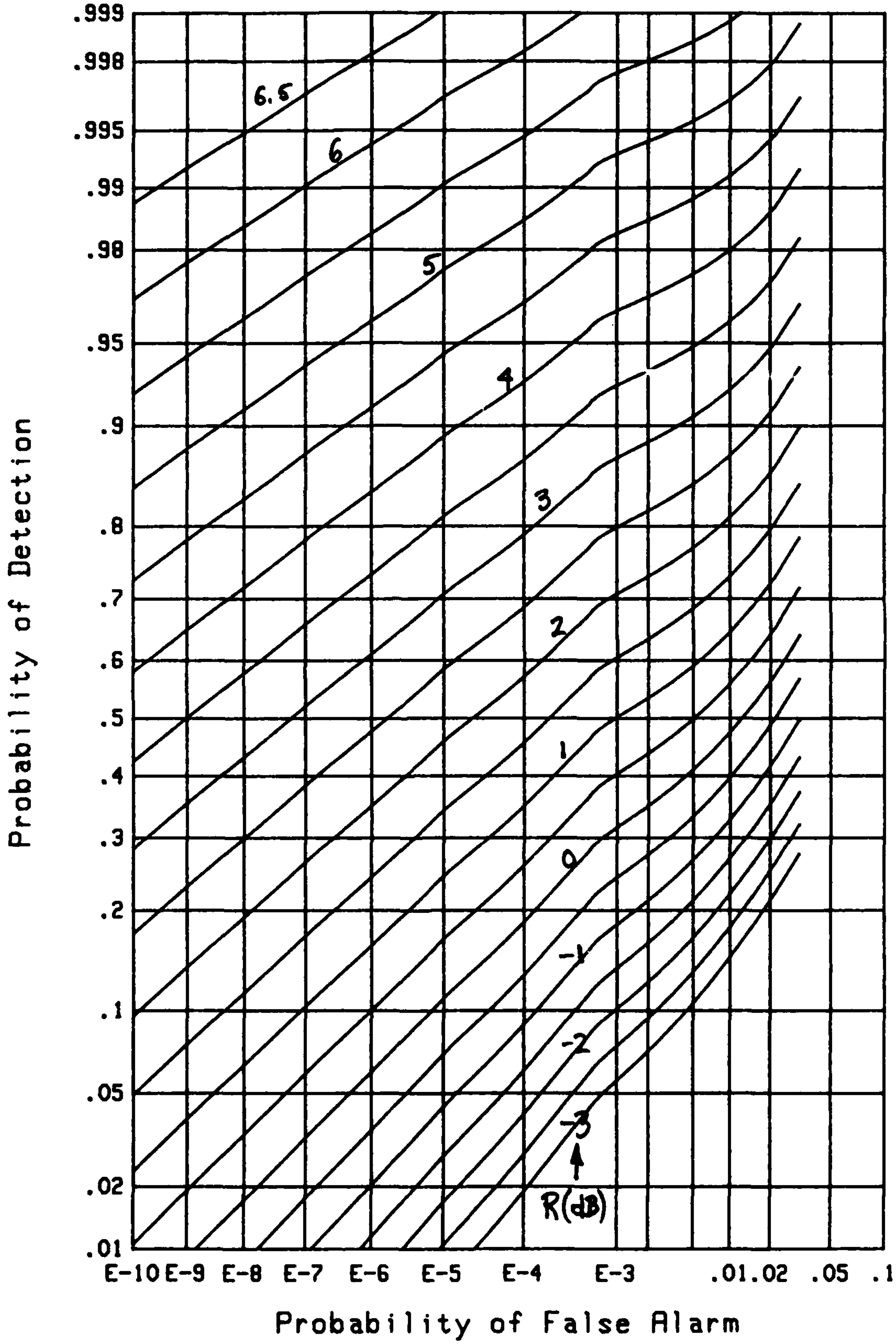

Figure 27. ROC for N=32, F=.001

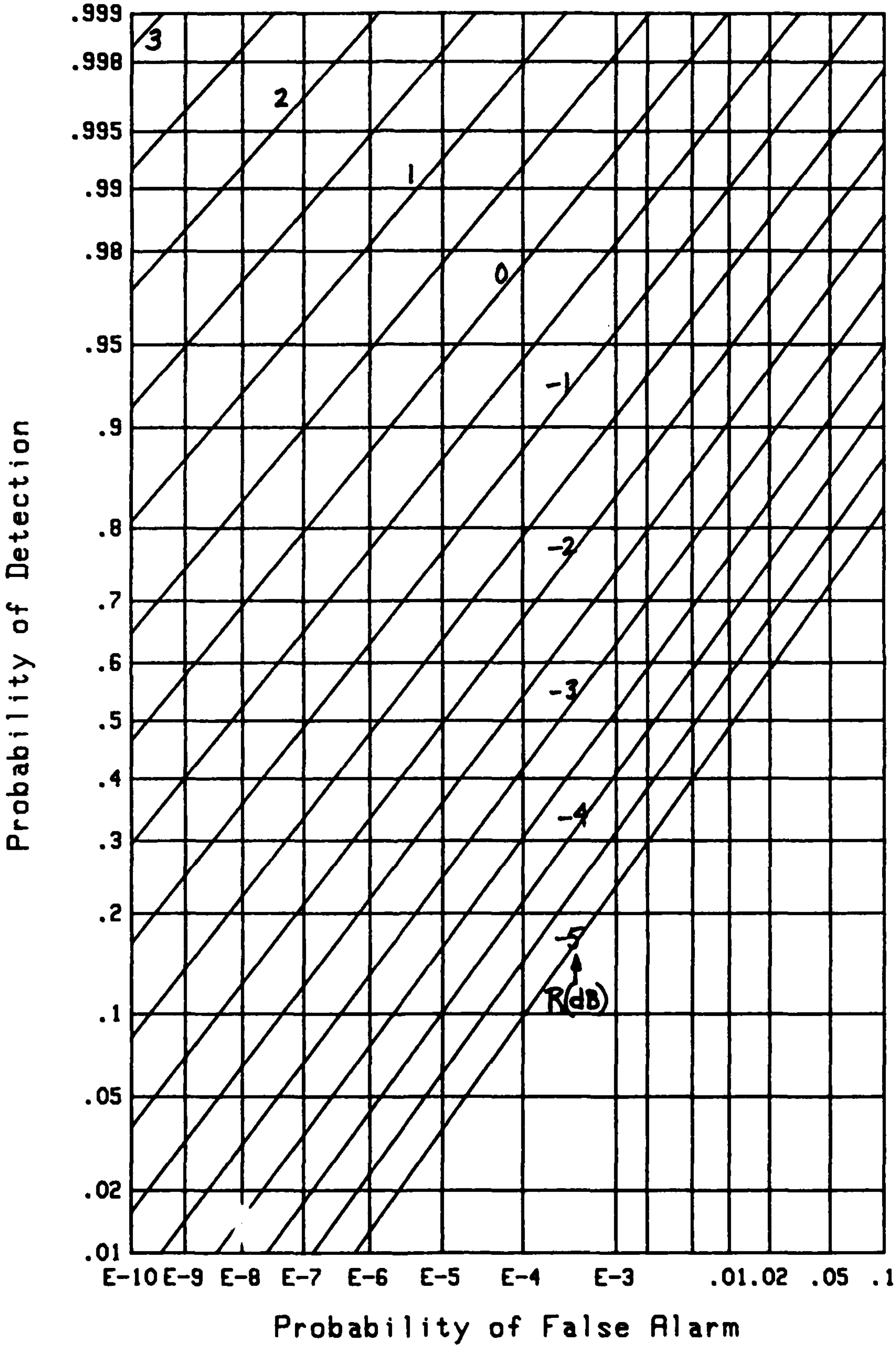

Figure 28. ROC for N=64, F=1.

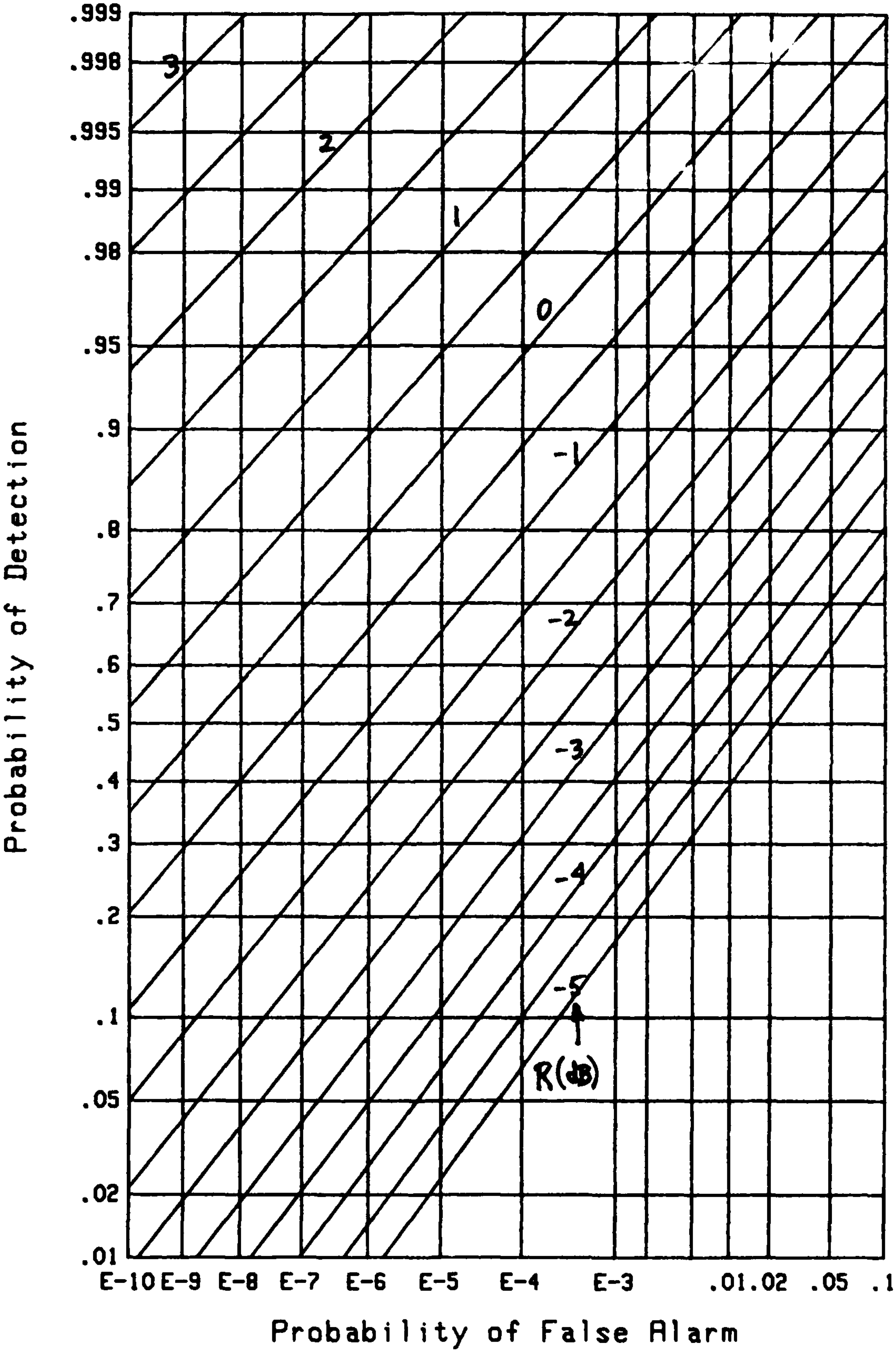

Figure 29. ROC for N=64, F=.1

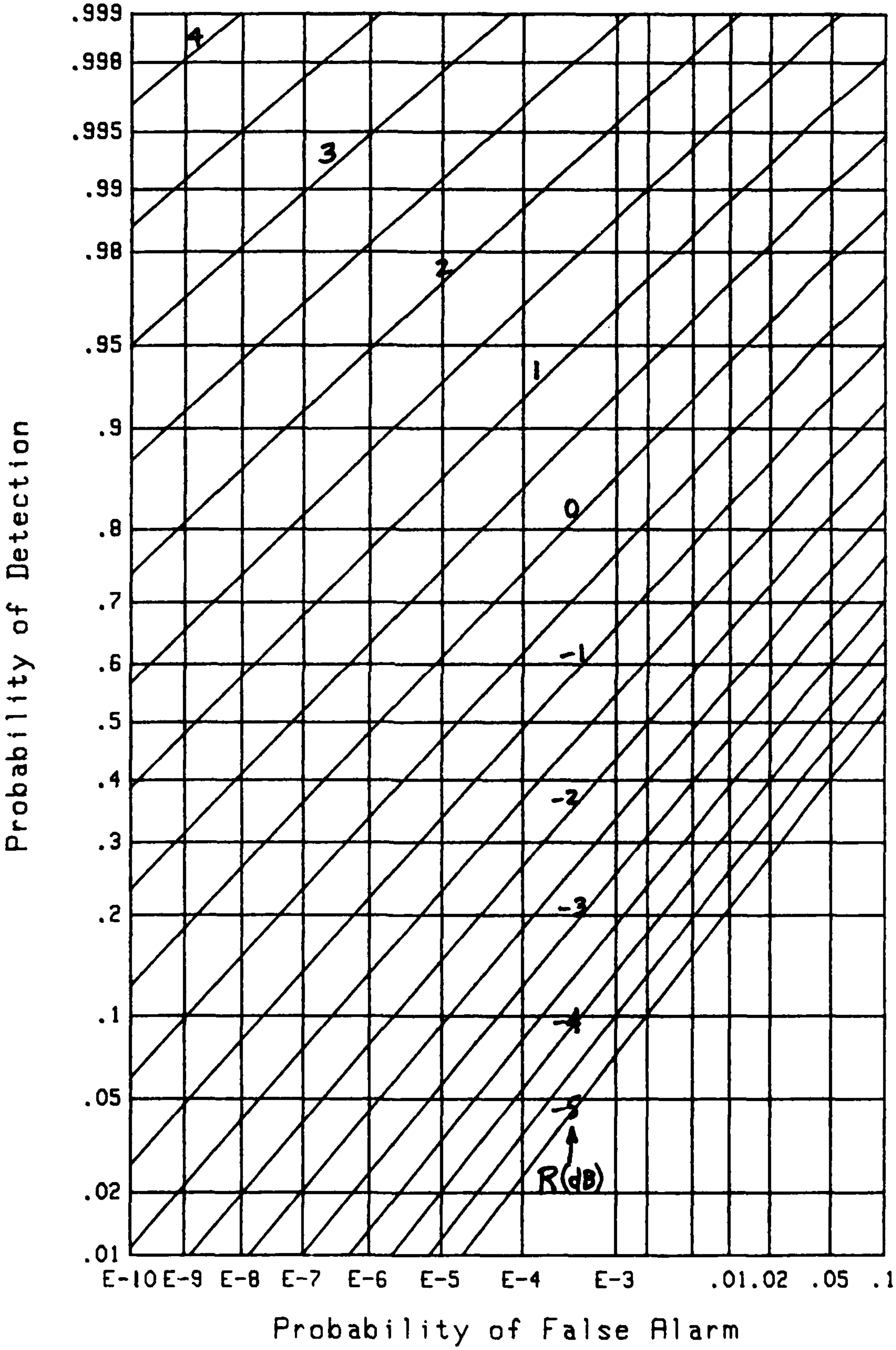

Figure 30. ROC for N=64, F=.01

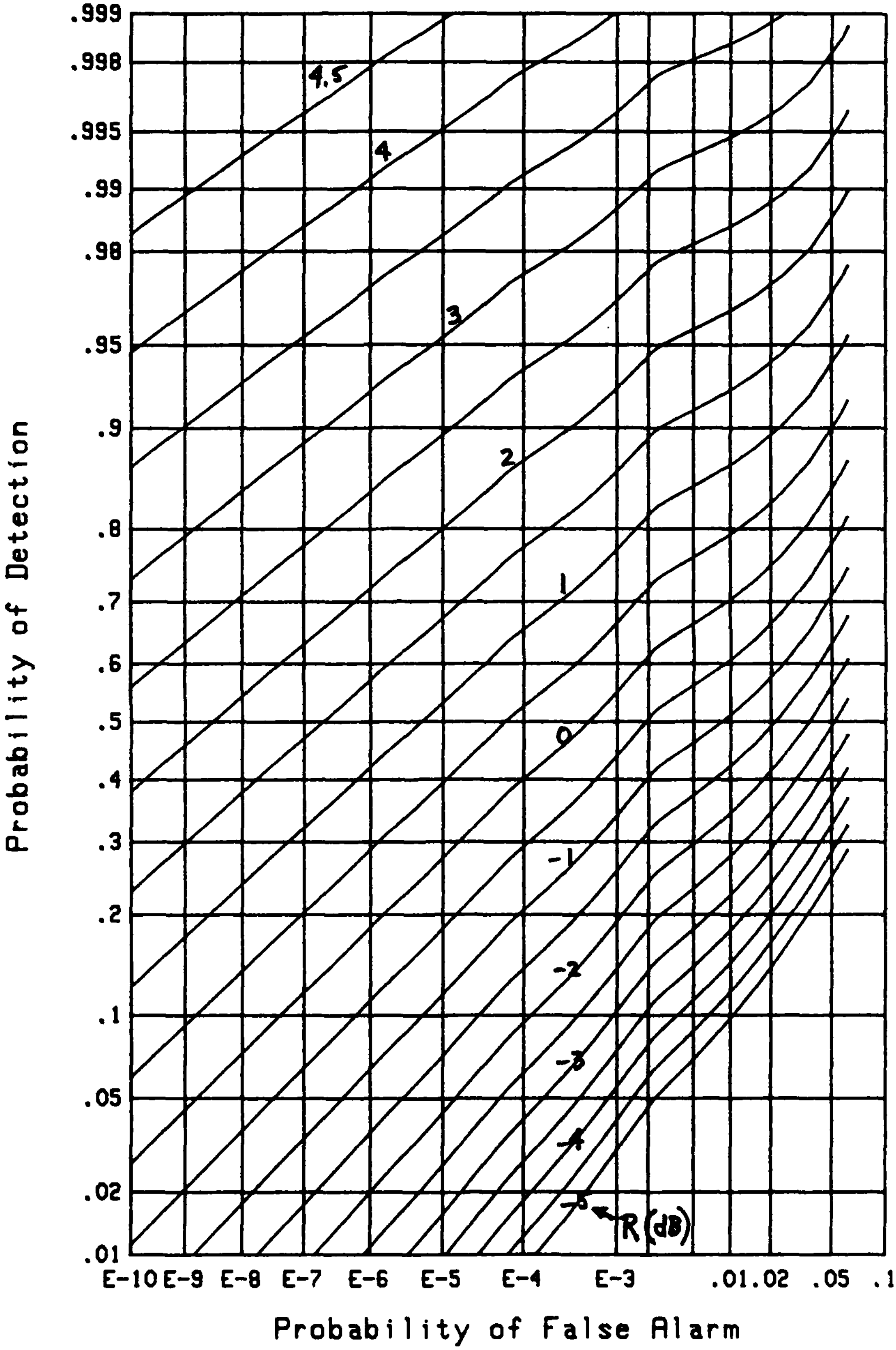

Figure 31. ROC for N=64, F=.001

SUMMARY

The cost of suppressing the low-level outputs of the detected squared-envelopes is generally minimal, unless the number of channels becomes very large. This conclusion has been drawn only for the example where these squared-envelopes have an exponential probability density function for both the noise-only as well as the signal-plus-noise cases. It should also be checked out for other candidate forms of probability density functions besides exponential.

One line of reasoning that makes this conclusion more acceptable is that it is only the larger outputs from the detectors that are going to lead to positive statements about signal presence. Thus, suppression of the smaller outputs should be inconsequential, at least for few channels. However, for a large number of channels, the sum of many nonzero low-level quantities may add up to a significant value and affect an occasional detection decision.

APPENDIX A. PROGRAM FOR RECEIVER OPERATING CHARACTERISTICS

```
10  ! GENERATE PD VS PF; COMBINER WITH DEAD ZONE IN EACH CHANNEL, TR8595
20    N=64                    !  NUMBER OF CHANNELS; N>=1
30    F=.001                  !  FRACTION OF DATA PASSED; 0.<F<=1.
40    DIM T(100)              !  THRESHOLD VALUES
50    COM Pf(100),Pd1(100),Pd2(100),Pd3(100),Pd4(100),Pd5(100)
60    COM Pd6(100),Pd7(100),Pd8(100),Pd9(100),Pd10(100),Pd11(100)
70    COM Pd12(100),Pd13(100),Pd14(100),Pd15(100),Pd16(100),Pd17(100)
80    COM Pd18(100),Pd19(100),Pd20(100)
90    DOUBLE N,I,J            !  INTEGERS
100   T=.01
110   T=T+.01
120   Pf=FNPf(T,F,N)
130     IF Pf>.1 THEN 110
140   T1=MAX(T-.01,.01)
150   T=T+.01
160   Pf=FNPf(T,F,N)
170     IF Pf>1E-10 THEN 150
180   T2=T
190   Delt=(T2-T1)/100.
200   FOR I=0 TO 100
210   T=T1+Delt*I
220   T(I)=T                  !  THRESHOLD VALUES
230   Pf(I)=FNPf(T,F,N)       !  FALSE ALARM PROBABILITIES
240   NEXT I
250     R1db=-5               !  STARTING SIGNAL-TO-NOISE RATIO (dB)
260     Delr=.5               !  INCREMENT IN SNR (dB)
270   FOR J=1 TO 20
280     Rdb=R1db+(J-1)*Delr   !  SIGNAL-TO-NOISE RATIO PER CHANNEL (dB)
290   R=10.^(.1*Rdb)          !  POWER RATIO
300   FOR I=0 TO 100
310   T=T(I)
320   Pd=FNPd(R,T,F,N)        !  DETECTION PROBABILITIES
330   IF J=1 THEN Pd1(I)=Pd
340   IF J=2 THEN Pd2(I)=Pd
350   IF J=3 THEN Pd3(I)=Pd
360   IF J=4 THEN Pd4(I)=Pd
370   IF J=5 THEN Pd5(I)=Pd
380   IF J=6 THEN Pd6(I)=Pd
390   IF J=7 THEN Pd7(I)=Pd
400   IF J=8 THEN Pd8(I)=Pd
410   IF J=9 THEN Pd9(I)=Pd
420   IF J=10 THEN Pd10(I)=Pd
430   IF J=11 THEN Pd11(I)=Pd
440   IF J=12 THEN Pd12(I)=Pd
450   IF J=13 THEN Pd13(I)=Pd
460   IF J=14 THEN Pd14(I)=Pd
470   IF J=15 THEN Pd15(I)=Pd
480   IF J=16 THEN Pd16(I)=Pd
490   IF J=17 THEN Pd17(I)=Pd
500   IF J=18 THEN Pd18(I)=Pd
510   IF J=19 THEN Pd19(I)=Pd
520   IF J=20 THEN Pd20(I)=Pd
530   NEXT I
540   NEXT J
```

```
550     FOR I=0 TO 100
560     Pf(I)=FNInvphi(Pf(I))
570     Pd1(I)=FNInvphi(Pd1(I))
580     Pd2(I)=FNInvphi(Pd2(I))
590     Pd3(I)=FNInvphi(Pd3(I))
600     Pd4(I)=FNInvphi(Pd4(I))
610     Pd5(I)=FNInvphi(Pd5(I))
620     Pd6(I)=FNInvphi(Pd6(I))
630     Pd7(I)=FNInvphi(Pd7(I))
640     Pd8(I)=FNInvphi(Pd8(I))
650     Pd9(I)=FNInvphi(Pd9(I))
660     Pd10(I)=FNInvphi(Pd10(I))
670     Pd11(I)=FNInvphi(Pd11(I))
680     Pd12(I)=FNInvphi(Pd12(I))
690     Pd13(I)=FNInvphi(Pd13(I))
700     Pd14(I)=FNInvphi(Pd14(I))
710     Pd15(I)=FNInvphi(Pd15(I))
720     Pd16(I)=FNInvphi(Pd16(I))
730     Pd17(I)=FNInvphi(Pd17(I))
740     Pd18(I)=FNInvphi(Pd18(I))
750     Pd19(I)=FNInvphi(Pd19(I))
760     Pd20(I)=FNInvphi(Pd20(I))
770     NEXT I
780     CALL Plot
790     END
800     !
810     DEF FNInvphi(X)         !  AMS 55, 26.2.23
820     IF X=.5 THEN RETURN 0.
830     P=MIN(X,1.-X)
840     T=-LOG(P)
850     T=SQR(T+T)
860     P=1.+T*(1.432788+T*(.189269+T*.001308))
870     P=T-(2.515517+T*(.802853+T*.010328))/P
880     IF X<.5 THEN P=-P
890     RETURN P
900     FNEND
910     !
920     DEF FNE(U,DOUBLE N)     !  N>=0
930     DOUBLE K                !  INTEGER
940     IF U<=0. THEN RETURN 1.
950     S=T=EXP(-U)
960     IF N=0 THEN RETURN S
970     FOR K=1 TO N
980     T=T*U/K
990     S=S+T
1000    NEXT K
1010    RETURN S
1020    FNEND
1030    !
```

```
1040    DEF FNPf(T,F,DOUBLE N) !  FALSE ALARM PROB.    T>=0,0<F<=1,N>=1
1050    DOUBLE Ns,N1           !  INTEGERS
1060    IF F<1. THEN 1090
1070    Pf=FNE(T,N-1)
1080    RETURN Pf
1090    L=-LOG(F)
1100    N1=N+1
1110    F1=1.-F
1120    A=F/F1
1130    Tn=F1^N
1140    C=T
1150    Pf=0.
1160    FOR Ns=1 TO N
1170    C=C-L
1180    Tn=Tn*A*(N1-Ns)/Ns
1190    Pf=Pf+Tn*FNE(C,Ns-1)
1200    NEXT Ns
1210    RETURN Pf
1220    FNEND
1230    !
1240    DEF FNPd(R,T,F,DOUBLE N) ! DETECTION PROB.  R>=0,T>=0,0<F<=1,N>=1
1250    DOUBLE Ns,N1           !  INTEGERS
1260    As=1./(1.+R)           !  a
1270    IF F<1. THEN 1300
1280    Pd=FNE(As*T,N-1)
1290    RETURN Pd
1300    L=-LOG(F)
1310    N1=N+1
1320    B=EXP(-As*L)
1330    B1=1.-B
1340    A=B/B1
1350    Tn=B1^N
1360    C=T
1370    Pd=0.
1380    FOR Ns=1 TO N
1390    C=C-L
1400    Tn=Tn*A*(N1-Ns)/Ns
1410    Pd=Pd+Tn*FNE(As*C,Ns-1)
1420    NEXT Ns
1430    RETURN Pd
1440    FNEND
1450    !
```

```
1460    SUB Plot   !  PLOT PD VS PF ON NORMAL PROBABILITY PAPER
1470    COM Pf(*),Pd1(*),Pd2(*),Pd3(*),Pd4(*),Pd5(*)
1480    COM Pd6(*),Pd7(*),Pd8(*),Pd9(*),Pd10(*),Pd11(*)
1490    COM Pd12(*),Pd13(*),Pd14(*),Pd15(*),Pd16(*),Pd17(*)
1500    COM Pd18(*),Pd19(*),Pd20(*)
1510    DIM A$[30],B$[30],C$[31]
1520    DIM Xlabel$(1:30),Ylabel$(1:30)
1530    DIM Xcoord(1:30),Ycoord(1:30)
1540    DIM Xgrid(1:30),Ygrid(1:30)
1550    DOUBLE N,Lx,Ly,Nx,Ny,I   !  INTEGERS
1560    !
1570    A$="Probability of False Alarm"
1580    B$="Probability of Detection"
1590      C$="Figure     ROC for N=64, F=.001"
1600    !
1610    Lx=12
1620    REDIM Xlabel$(1:Lx),Xcoord(1:Lx)
1630    DATA E-10,E-9,E-8,E-7,E-6,E-5,E-4,E-3,.01,.02,.05,.1
1640    READ Xlabel$(*)
1650    DATA 1E-10,1E-9,1E-8,1E-7,1E-6,1E-5,1E-4,.001,.01,.02,.05,.1
1660    READ Xcoord(*)
1670    !
1680    Ly=18
1690    REDIM Ylabel$(1:Ly),Ycoord(1:Ly)
1700    DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1710    DATA .95,.98,.99,.995,.998,.999
1720    READ Ylabel$(*)
1730    DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1740    DATA .95,.98,.99,.995,.998,.999
1750    READ Ycoord(*)
1760    !
1770    Nx=14
1780    REDIM Xgrid(1:Nx)
1790    DATA 1E-10,1E-9,1E-8,1E-7,1E-6,1E-5,1E-4
1800    DATA .001,.002,.005,.01,.02,.05,.1
1810    READ Xgrid(*)
1820    !
1830    Ny=18
1840    REDIM Ygrid(1:Ny)
1850    DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1860    DATA .95,.98,.99,.995,.998,.999
1870    READ Ygrid(*)
1880    !
1890    FOR I=1 TO Lx
1900    Xcoord(I)=FNInvphi(Xcoord(I))
1910    NEXT I
1920    FOR I=1 TO Ly
1930    Ycoord(I)=FNInvphi(Ycoord(I))
1940    NEXT I
1950    FOR I=1 TO Nx
1960    Xgrid(I)=FNInvphi(Xgrid(I))
1970    NEXT I
1980    FOR I=1 TO Ny
1990    Ygrid(I)=FNInvphi(Ygrid(I))
2000    NEXT I
```

```
2010    X1=Xgrid(1)
2020    X2=Xgrid(Nx)
2030    Y1=Ygrid(1)
2040    Y2=Ygrid(Ny)
2050    GINIT 200./260.                         !  VERTICAL PAPER
2060    PLOTTER IS 505,"HPGL"
2070    PRINTER IS 505
2080      PRINT "VS2"
2090    LIMIT PLOTTER 505,0.,200.,0.,260.       !  1 GDU = 2 mm
2100    VIEWPORT 22.,85.,19.,122.
2110    WINDOW X1,X2,Y1,Y2
2120    FOR I=1 TO Nx
2130    MOVE Xgrid(I),Y1
2140    DRAW Xgrid(I),Y2
2150    NEXT I
2160    FOR I=1 TO Ny
2170    MOVE X1,Ygrid(I)
2180    DRAW X2,Ygrid(I)
2190    NEXT I
2200    LDIR 0
2210    CSIZE 2.3,.5
2220    LORG 5
2230    Y=Y1-(Y2-Y1)*.02
2240    FOR I=1 TO Lx
2250    MOVE Xcoord(I),Y
2260    LABEL Xlabel$(I)
2270    NEXT I
2280    CSIZE 3.,.5
2290    MOVE .5*(X1+X2),Y1-.06*(Y2-Y1)
2300    LABEL A$
2310    MOVE .5*(X1+X2),Y1-.1*(Y2-Y1)
2320    LABEL C$
2330    CSIZE 2.3,.5
2340    LORG 8
2350    X=X1-(X2-X1)*.01
2360    FOR I=1 TO Ly
2370    MOVE X,Ycoord(I)
2380    LABEL Ylabel$(I)
2390    NEXT I
2400    LDIR PI/2.
2410    CSIZE 3.,.5
2420    LORG 5
2430    MOVE X1-.15*(X2-X1),.5*(Y1+Y2)
2440    LABEL B$
2450    PENUP
```

```
2460    PLOT Pf(*),Pd1(*)
2470    PENUP
2480    PLOT Pf(*),Pd2(*)
2490    PENUP
2500    PLOT Pf(*),Pd3(*)
2510    PENUP
2520    PLOT Pf(*),Pd4(*)
2530    PENUP
2540    PLOT Pf(*),Pd5(*)
2550    PENUP
2560    PLOT Pf(*),Pd6(*)
2570    PENUP
2580    PLOT Pf(*),Pd7(*)
2590    PENUP
2600    PLOT Pf(*),Pd8(*)
2610    PENUP
2620    PLOT Pf(*),Pd9(*)
2630    PENUP
2640    PLOT Pf(*),Pd10(*)
2650    PENUP
2660    PLOT Pf(*),Pd11(*)
2670    PENUP
2680    PLOT Pf(*),Pd12(*)
2690    PENUP
2700    PLOT Pf(*),Pd13(*)
2710    PENUP
2720    PLOT Pf(*),Pd14(*)
2730    PENUP
2740    PLOT Pf(*),Pd15(*)
2750    PENUP
2760    PLOT Pf(*),Pd16(*)
2770    PENUP
2780    PLOT Pf(*),Pd17(*)
2790    PENUP
2800    PLOT Pf(*),Pd18(*)
2810    PENUP
2820    PLOT Pf(*),Pd19(*)
2830    PENUP
2840    PLOT Pf(*),Pd20(*)
2850    PENUP
2860    BEEP 500,2
2870    PAUSE
2880    PRINTER IS CRT
2890    PLOTTER 505 IS TERMINATED
2900    SUBEND
```

REFERENCES

1. A. H. Nuttall, **Accurate Efficient Evaluation of Cumulative or Exceedance Probability Distributions Directly From Characteristic Functions**, NUSC Technical Report 7023, Naval Underwater Systems Center, New London, CT, 1 October 1983.

2. A. H. Nuttall, **Operating Characteristics of Cross-Correlator With or Without Sample Mean Removal**, NUSC Technical Report 7045, Naval Underwater Systems Center, New London, CT, 14 August 1984.

3. A. H. Nuttall, **Evaluation of Densities and Distributions via Hermite and Generalized Laguerre Series Employing High-Order Expansion Coefficients Determined Recursively via Moments or Cumulants**, NUSC Technical Report 7377, Naval Underwater Systems Center, New London, CT, 28 February 1985.

4. **Handbook of Mathematical Functions**, U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series No. 55, U. S. Government Printing Office, Washington, DC, June 1964.

5. A. H. Nuttall and A. F. Magaraci, **Signal-to-Noise Ratios Required for Short-Term Narrowband Detection of Gaussian Processes**, NUSC Technical Report 4417, Naval Underwater Systems Center, New London, CT, 20 October 1972.

INITIAL DISTRIBUTION LIST

Addressee	No. of Copies
ADMIRALTY RESEARCH ESTABLISHMENT, London, England (Dr. L. Lloyd)	1
ADMIRALTY UNDERWATER WEAPONS ESTABLISHMENT, Dorset, England	1
APPLIED PHYSICS LAB, JOHN HOPKINS (John C. Stapleton)	1
APPLIED PHYSICS LAB, U. WASHINGTON (C. Eggan)	1
APPLIED RESEARCH LAB, PENN STATE,(Frank W. Symons)	1
APPLIED RESEARCH LAB, U. TEXAS (Dr. M. Frazer)	1
APPLIED SEISMIC GROUP, Cambridge, MA (Richard Lacoss)	1
A & T, Stonington, Ct (H. Jarvis)	1
ASTRON RESEARCH & ENGINEERING, Santa Monica, CA (Dr. Allen Piersol)	1
BBN, Arlington, Va. (Dr. H. Cox)	1
BBN, Cambridge, MA (H. Gish)	1
BBN, New London, Ct. (Dr. P. Cable)	1
BELL COMMUNICATIONS RESEARCH, Morristown, NJ (J. Kaiser and D. Sunday (Library)	2
BENDAT, Julius Dr., Los Angeles, CA	1
BLEINSTEIN, Norman Dr., Denver, CO	1
BROWN UNIV, Providence, RI (Documents Library)	1
CANBERRA COLLEGE OF ADV. EDUC, BELCONNEN, A.C.T. Australia (P. Morgan)	1
COAST GUARD ACADEMY, New London, CT (Prof. J. Wolcin)	1
COAST GUARD R & D, Groton, CT (Library)	1
COGENT SYSTEMS, INC, (J. Costas)	1
COHEN, Leon Dr., Bronx, NY	1
CONCORDIA UNIVERSITY H-915-3, Montreal, Quebec Canada (Prof. Jeffrey Krolik)	1
CNO (NOP-098)	1
CNR-OCNR-00, 10, 12, 13, 20	5
DALHOUSIE UNIV., Halifax, Nova Scotia, Canada (Dr. B. Ruddick)	1
DAVID W. TAYLOR RESEARCH CNTR, Annapolis, MD (P. Prendergast, Code 2744)	1
DARPA, Arlington, VA (A. Ellinthorpe)	1
DEFENCE RESEARCH CENTER, Adeliade, Australia (Library	1
DEFENCE RESEARCH ESTAB. ATLANTIC, Dartmouth, Nova Scotia (Library)	1
DEFENCE RESEARCH ESTAB. PACIFIC, Victoria, Canada (Dr. D. Thomson)	1
DEFENCE SCIENTIFIC ESTABLISHMENT, MINISTRY OF DEFENCE, Auckland, New Zealand (Dr. L. Hall)	1
DEFENSE SYSTEMS, INC, Mc Lean, VA (Dr. G. Sebestyen)	1
DIA	1
DIAGNOSTIC/RETRIEVAL SYSTEMS, INC, Tustin, CA. (J. Williams)	1
DTIC	1
DTRC	1
DREXEL UNIV, (Prof. S. Kesler)	1
EDO CORP, College Point, NY (M. Blanchard)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
EG&G, Manassas, VA (D. Frohman)	1
GENERAL ELECTRIC CO, Moorestown, NJ (Dr. Mark Allen 108-102)	1
GENERAL ELECTRIC CO., Philadelphia, PA (T. J. McFall)	1
GENERAL ELECTRIC CO, Pittsfield, MA (R. W. Race)	1
GENERAL ELECTRIC CO, Syracuse, NY (J. L. Rogers, Dr. A. M. Vural and D. Winfield)	3
HAHN, Wm, Wash, DC	1
HARRIS SCIENTIFIC SERVICES, Dobbs Ferry, NY (B. Harris)	1
HARVARD UNIVERSITY, Gordon McKay Library	1
HONEYWELL ENGR SERV CNTR, Poulsbro, WA (C. Schmid)	1
HUGHES AIRCRAFT, Fullerton, CA (S. Autrey)	1
HUGHES AIRCRAFT, Buena Park, CA (T. Posch)	1
IBM, Manassas, VA (G. Demuth)	1
INDIAN INSTITUTE OF TECHNOLOGY, Madras, India (Dr. K. M. M. Prabhu)	1
INTERSTATE ELECTRONICS CORP, Anaheim, CA (R. Nielsen, 8011)	1
JOHNS HOPKINS UNIV, Laurel, MD (J. C. Stapleton)	1
KILDARE CORP, New London, CT (Dr. R. Mellen)	1
LINCOM CORP., Northboro, MA (Dr. T. Schonhoff)	1
MAGNAVOX ELEC SYSTEMS CO, Ft. Wayne, IN (R. Kenefic)	1
MARINE BIOLOGICAL LAB, Woods Hole, MA	1
MARINE PHYSICAL LABORATORY SCRIPPS	1
MASS. INSTITUTE OF TECHNOLOGY (Prof. A. Baggaroer, Barker Engineering Library)	2
MBS SYSTEMS, Norwalk, CT (A. Winder)	1
MIDDLETON, DAVID, NY, NY	1
NADC (5041, M. Mele)	1
NAIR-03	1
NASH, Harold E., Quaker Hill, CT	1
NATIONAL RADIO ASTRONOMY OBSERVATORY, Charlottesville, VA (F. Schwab)	1
NATIONAL SECURITY AGENCY, FT. Meade, MD (Dr. James R. Maar, R51)	1
NATO SACLANT ASW RESEARCH CENTRE, APO NY, NY (Library, R. E. Sullivan and G. Tacconi)	3
NCSC	1
NEPRF	1
NORDA	1
NRL UND SOUND REF DET, Orlando, FL	1
NAVAL INTELLIGENCE COMMAND	1
NAVAL INTELLIGENCE SUPPORT CENTER	1
NAVAL OCEAN SYSTEMS CENTER, San Diego, CA (James M. Alsup, Code 635)	1
NAVAL OCEANOGRAPHY OFFICE	1
NAVAL SYSTEMS DIV., SIMRAD SUBSEA A/S, Norway (E. B. Lunde)	1
NICHOLS RESEARCH CORP., Wakefield, MA (T. Marzetta)	1
NORDA (Dr. B. Adams)	1
NORDA (Code 345) N STL Station, MS (R. Wagstaff)	1
NORTHEASTERN UNIV. Boston, MA (Prof. C. L. Nikias)	1
NORWEGIAN DEFENCE RESEARCH EST, Norway (Dr J. Glattetre)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
NOSC, (James M. Alsup, Code 635, C. Sturdevant; 73, J. Lockwood, F. Harris, 743, R. Smith; 62, R. Thuleen)	6
NPRDC	1
NPS, Monterey, CA (C. W. Therrien, Code 62 Ti)	2
NRL, Washington, DC (Dr. J. Buccaro, Dr. E. Franchi, Dr. P. Abraham, Code 5132, A. A. Gerlach, W. Gabriel (Code 5370), and N. Yen (Code 5130)	6
NRL, Arlington, VA (N. L. Gerr, Code 1111)	1
NSWC	1
NSWC DET Ft. Lauderdale	1
NSWC WHITE OAK LAB	1
NUSC DET TUDOR HILL	1
NUSC DET WEST PALM BEACH (Dr. R. Kennedy Code 3802)	1
NWC	1
ORI CO, INC, New London, CT (G. Assard)	1
ORINCON CORP., Columbia, MD (S. Larry Marple)	1
PAPOUTSANIS, P. D., Athens, Greece	1
PENN STATE UNIV., State College, PA (F. Symons)	1
PIERSOLL ENGR CO, Woodland Hills, CA (Dr. Allen G. Piersol)	1
POHLER, R., Austin, TX	1
POLETTI, Mark A., Acoustics Research Centre, School of Architecture, Univ. of Auckland, Auckland, New Zealand	1
PROMETHEUS, INC, Sharon, MA (Dr. J. Byrnes)	1
PROMETHEUS INC, Newport, RI (Michael J. Barrett)	1
PRICE, Robert Dr. Lexington, MA	1
PURDUE UNIV, West Lafayette, IN (N. Srinivasa)	1
RAISBECK, Dr. Gordon, Portland, ME	1
RAN RESEARCH LAB, Darlinghurst, Australia	1
RAYTHEON CO, Portsmouth, RI (J. Bartram, R. Connor) and S. S. Reese)	3
RICHTER, W., Annandale, VA.	1
ROCKWELL INTERNATIONAL CORP, Anaheim, CA (L. Einstein and Dr. D. Elliott)	2
ROYAL MILITARY COLLEGE OF CANADA, (Prof. Y. Chan)	1
RUTGERS UNIV., Piscataway, NJ (Prof. S. Orfanidis)	1
RCA CORP, Moorestown, NJ (H. Upkowitz)	1
SACLANT UNDERSEA RESEARCH CENTRE, APO NY NY (Dr. John Ianniello, Dr. S. Stergiopolous and Giorgio Tacconi, Library	4
SAIC, Falls Church, VA (Dr. P. Mikhalevsky)	1
SAIC, New London, CT (Dr. F. Dinapoli)	1
SANDIA NATIONAL LABORATORY (J. Claasen)	1
SCHULKIN, Dr. Morris, Potomac, MD	1
SEA-00, 63, 63X	3
SIMON FRASER UNIV, British Columbia, Canada (Dr. Edgar Velez)	1
SONAR SURVEILLANCE GROUP, Darlinghurst, Australia	1
SOUTHEASTERN MASS. UNIV (Prof. C. H. Chen)	1
SPAWARS-00, 04, 005, PD-80 and PMW-181	5
SPERRY CORP, Great Neck, NY	1
STATE UNIV. OF NY AT STONY BROOK (Prof. M. Barkat)	1
TEL-AVIV UNIV, Tel-Aviv, Israel (Prof. E. Winstein)	1
TOYON RESEARCH CORP, Goleta, CA (M. Van Blaricum)	1
TRACOR, INC, Austin, TX (Dr. T Leih and J. Wilkinson)	2
TRW FEDERAL SYSTEMS GROUP, Fairfax, VA (R. Prager)	1
UNITED ENGINEERING CENTER, Engr. Societies Library, NY, NY	1
UNIV. OF AUCKLAND, New Zealand (Dr. Murray D. Johns)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
UNIV. OF ALBERTA, Edmonton, Alberta, CANADA (K. Yeung)	1
UNIV OF CA, San Diego, CA (Prof. C. Helstrom)	1
UNIV OF COLORADO, Boulder, CO (Prof. L. Scharf)	1
UNIV. OF CT, Storrs, CT. (Library and Prof. C. Knapp)	2
UNIV OF FLA, Gainesville, FL (D. Childers)	1
UNIV OF ILLINOIS, Urbana, IL 61801 (Dr. Douglas L. Jones)	1
UNIV OF MICHIGAN, Ann Arbor, MI (EECS Bldg. North Campus))	1
UNIV. OF MINN, Minneapolis, Mn (Prof. M. Kaveh)	1
UNIV. OF NEWCASTLE, Newcastle, NSW, Canada (Prof. A. Cantoni)	1
UNIV. OF QUEENSLAND, St. Lucia, Queensland 4067, Australia (Dr. Boualem Boashash)	1
UNIV. OF RI, Kingston, RI (Prof. G. F. Boudreaux-Bartels, Library, Prof. S. Kay, and Prof. D. Tufts)	4
UNIV. OF ROCHESTER, Rochester, NY (Prof. E. Titlebaum)	1
UNIV. OF SOUTHERN CA., LA. (Prof. William C. Lindsey, Dr. Andreas Polydoros, PHE 414)	2
UNIV. OF STRATHCLYDE, ROYAL COLLEGE, Glasgow, Scotland (Prof. T. Durrani)	1
UNIV. OF TECHNOLOGY, Loughborough, Leicestershire, England (Prof. J. Griffiths)	1
UNIV. OF WASHINGTON, Seattle (Prof. D. Lytle)	1
URICK, ROBERT, Silver Springs, MD	1
US AIR FORCE, Maxwell AF Base, AL (Library)	1
VAN ASSELT, Henrik, USEA S.P.A., La Spezia, Italy	1
VILLANOVA UNIV, Villanova, PA (Prof. Moeness G. Amin)	1
WEAPONS SYSTEMS RESEARCH LAB, Adelaide, Australia	2
WERBNER, A., Medford, MA	1
WESTINGHOUSE ELEC. CORP, OCEANIC DIV, Annapolis, MD (Dr. H. Newman and Dr. H. L. Price)	2
WESTINGHOUSE ELEC. CORP, Waltham, MA (D. Bennett)	1
WILSON JAMES H., San Clemente, CA	1
WOODS HOLE OCEANOGRAPHIC INSTITUTION (Dr. R. Spindel and Dr. E. Weinstein, Library)	3
YALE UNIV. (Library, Prof. P. Schultheiss and Prof. F. Tuteur)	3